FREQUENCY RESPONSE ANALYSIS AND DESIGN OF SINGLE-VALUED NONLINEAR SYSTEMS USING THE PARAMETER PLANE

by

John Philip Davis

United States Naval Postgraduate School

THESIS

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Frequency Response Analysis and Design
of
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Using the Parameter Plane

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John Philip Davis
Lieutenant(junior grade), United States Navy
B.S., United States Naval Academy, 1968

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ABSTRACT

Frequency response techniques are a valuable tool in the analysis and synthesis of linear systems. Extension of these techniques is made to analyze and design systems with a single-valued nonlinear element. The relationship between the characteristics of a nonlinear element and the frequency of the system is developed by simple calculations and a digital computer program.

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I. INTRODUCTION

Frequency response techniques are a valuable tool in the analysis and synthesis of linear systems. Reasons for this are, in part, the ease of presentation and manipulation of block diagrams and transfer functions, and in part, the ease of computation and interpretation of Bode diagrams and Nichol charts. It is therefore desirable to extend these techniques to include nonlinear systems. Block diagrams with nonlinear elements can be handled with the addition of some new rules. Handling transfer functions that include nonlinearities presents more of a problem. The nonlinear element's characteristics must be described in terms of frequency so that they can be treated mathematically with the linear part of the system. This thesis develops a technique which shows the relationship between the nonlinearity and the frequency response of the system. It then uses this relationship to perform analysis and synthesis of systems that contain nonlinearities.

Several assumptions are made and some conditions must be satisfied. The most important condition is that the nonlinearity be a single-valued nonlinearity. A single-valued nonlinearity is one that for any given input has only one output. This condition eliminates elements such as backlash, negative deficiency, two-position relay, relay with hysteresis, etc. By making this condition, a single-valued nonlinearity can be expressed as a variable gain device, with the gain a function of the signal into the device.

Two other main conditions are required. The first is the condition that the input to the system be a pure sine wave of constant amplitude. The second condition is that the input signal to the nonlinear component is a pure sine wave.

In order to describe the nonlinearity as a function of frequency, it must first be described as a variable gain device. This gain is a function of the signal amplitude into the element. Using the PARAMS program, a plot of the gain versus the signal amplitude can be made. Each curve in the resulting family is for a fixed frequency. With this plot the gain can be described as a function of frequency. Using this relationship between gain and frequency, the analysis can be completed on a frequency response plot with constant gain curves.

The analysis and synthesis techniques use the PARAMS program (reproduced in the computer print-out section) to make two types of plots. The first type of plot is a plot of nonlinear gain versus the signal amplitude into the nonlinear element with constant frequency. The second type of plot is a plot of the magnitude (or phase) of the open or closed loop function versus frequency. Each curve in the family is for a fixed value of gain for the nonlinear element.

A brief description of the operation of the PARAMS program is presented below. Let a transfer function be represented by,

$$T(s) = \frac{N(s)}{D(s)} \tag{1.1}$$

Where N(s) is of the form $A_o+A_1s+A_2s^2+\cdots A_{n-1}s^{n-1}+A_ns^n$ and D(s) is of the form $B_o+B_1s+B_2s^2+\cdots B_{m-1}s^{m-1}+B_ms^m$.

Each A_k and B_k can be represented by,

$$A_k = C_k + D_k\alpha + E_k\beta + F_k\alpha\beta \tag{1.2a}$$

$$B_k = G_k + H_k\alpha + I_k\beta + J_k\alpha\beta \tag{1.2b}$$

Where α and β are two variable parameters of the system. T(s) can be represented by,

$$T(s) = \frac{N_e(s) + N_o(s)}{D_e(s) + D_o(s)} \tag{1.3a}$$

and T(-s) by,

$$T(-s) = \frac{N_e(s) - N_o(s)}{D_e(s) - D_o(s)} \tag{1.3b}$$

Where $N_e(s)$ is the even order terms of the numerator of T(s), $N_o(s)$ is the odd order terms of the numerator of T(s), $D_e(s)$ is the even order terms of the denominator of T(s), and $D_o(s)$ is the odd order terms of the denominator of T(s).

The mangitude squared function is,

$$M^2 = [T(s)]\ [T(-s)] = \frac{[N_e(s)]^2 - [N_o(s)]^2}{[D_e(s)]^2 - [D_o(s)]^2} \qquad (1.4)$$

Each term of equation (1.4) is a even polynomial in s. Letting s = jw, equation (1.4) can be reduced to,[1]

$$M^2 = \frac{\sum_{k=0}^{N} A_k \omega^{2k}}{\sum_{k=0}^{m} B_k \omega^{2k}} \qquad (1.5)$$

Where A_k and B_k are given by equations (1.2a) and (1.2b).

By letting α or β be a constant, plots of the remaining variable parameter (β or α) versus magnitude can be made with frequency constant. In like manner, plots of magnitude versus frequency can be made with a constant parameter value.

Phase calculations can be made by utilizing the formula,

$$\Theta = \tan^{-1} \left[\frac{N_o(s) D_e(s) - N_e(s) D_o(s)}{N_e(s) D_e(s) - N_o(s) D_o(s)} \right] \qquad (1.6)$$

In this thesis α or β represents the variable nonlinear gain.

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[1] Glavis, G.O., Frequency Response in the Parameter Plane, Master's Thesis, Naval Postgraduate School, 1967.

II. INTRODUCTION TO NONLINEAR FREQUENCY RESPONSE ANALYSIS USING THE PARAMETER PLANE

A. PREFACE

As an introduction to the technique of nonlinear frequency response analysis, a fourth order system will be considered. The analysis is divided into two parts. One part is the analysis of the system with a nonlinear element in the forward path. The second part is the analysis of the system with a nonlinear element in the major feedback path. Each part will be analyzed for a saturation element and then analyzed for a dead zone element. All the assumptions made in the first chapter are valid for this example and all other examples used.

The second section of this chapter looks closer at the jump resonance effect that a nonlinear element can cause. The example taken is a simple second order servo with a saturation element in the forward path. The analysis techniques are the same in this example as were those in the first example of this chapter.

B. ANALYSIS OF THE FIRST EXAMPLE

The first example of this analysis is a fourth order system with a nonlinear element in either the forward path or the main feedback path. The fourth order system chosen for this example is a simple position servomechanism which is shown in Figure (2.1).

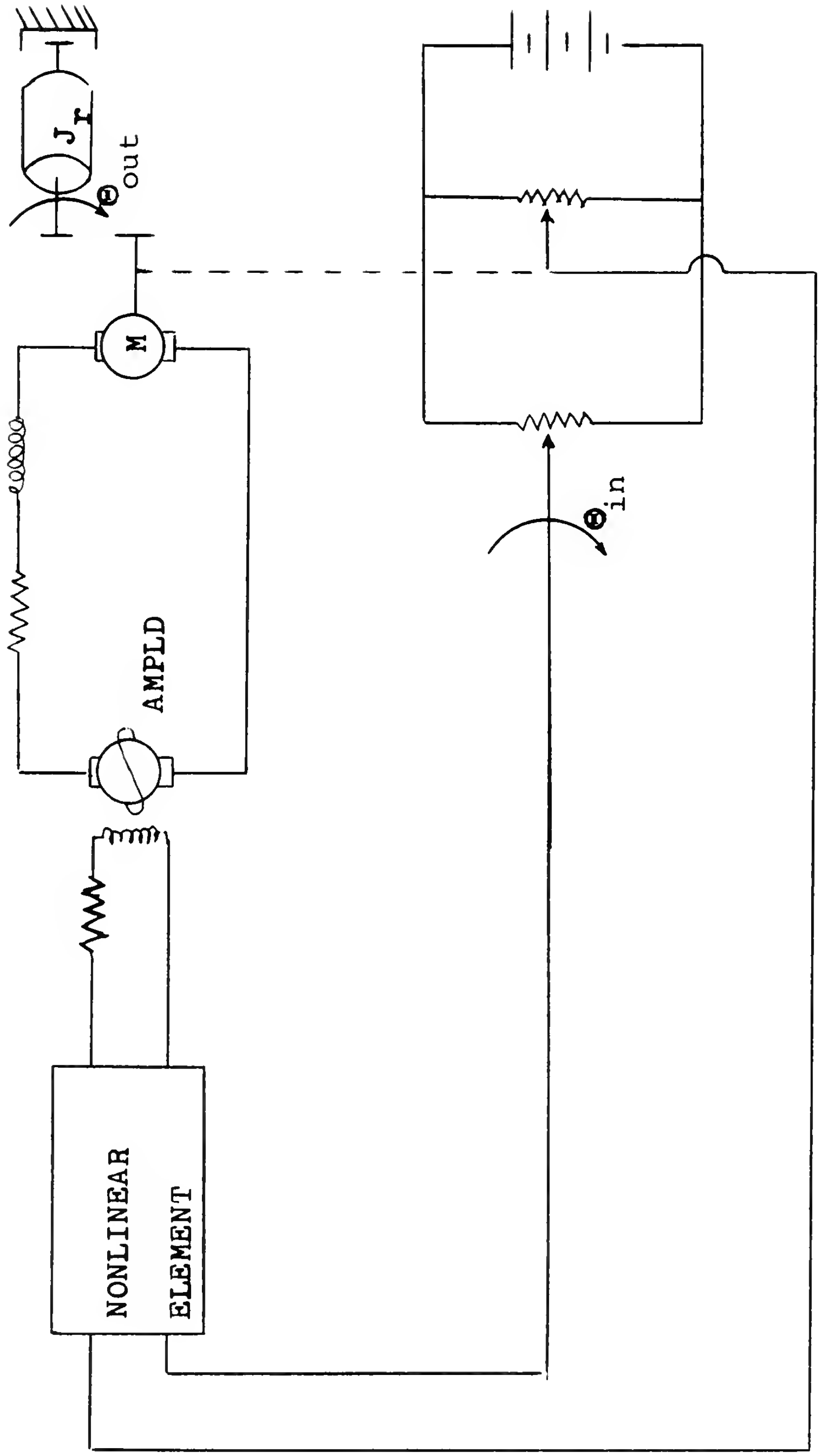

Figure 2.1 POSITION SERVOMECHANISM EXAMPLE OF CHAPTER 2

The transfer function of the amplidyne is,

$$G_a(s) = \frac{k_q}{(s + \frac{1}{\tau_f})(s + \frac{1}{\tau_q})} \tag{2.1}$$

where $\tau_f = \frac{l_f}{R_f}$ and $\tau_q = \frac{l_q}{R_q}$

The motor-load combination transfer function is,

$$G_m(s) = \frac{k_m}{s(s + \frac{1}{\tau_l})} \tag{2.2}$$

Where τ_l is the motor-load time constant and is pre-determined.

The error detector has a gain of k_e.

Putting the position servomechanism into block diagram form is done and is shown in Figure (2.2). The nonlinear element in the forward path is fed by the error signal Θ_e. The output signal, Θ_a feeds the nonlinear element in the main feedback path. These nonlinear elements are represented by blocks with their associated variable gain. α is the variable gain for the nonlinear element in the forward path and β is the variable gain for the nonlinear element in the main feedback path. The two nonlinear elements are not in the system at the same time. Analysis will be done for the nonlinearity in the forward path and then for the nonlinearity in the main feedback path.

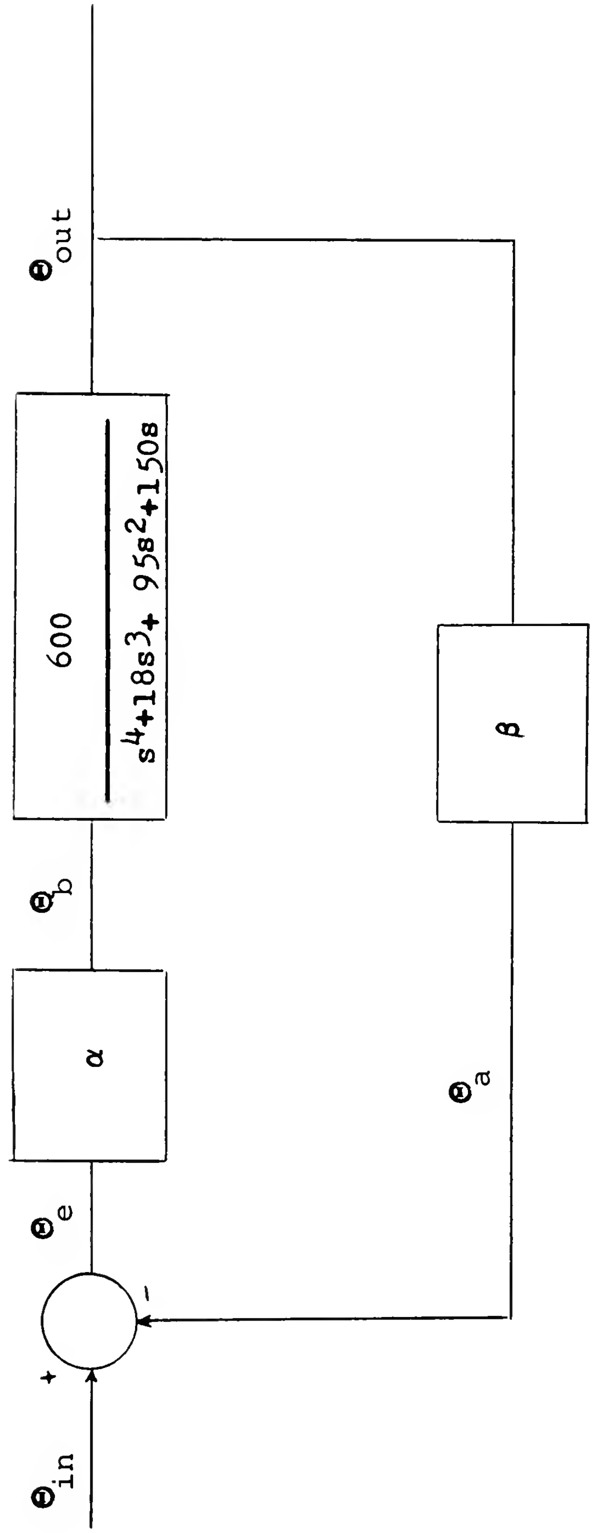

Figure 2.2 BLOCK DIAGRAM OF THE POSITION SERVOMECHANISM OF CHAPTER 2

Assume that $\frac{1}{\tau_f} = 10.0$, $\frac{1}{\tau_q} = 5$, $\frac{1}{\tau_1} = 3$, and $k_q k_m k_e = 600$. The characteristic equation now becomes,

$$s^4 + 18s^3 + 95s^2 + 150s + 600\alpha\beta = 0.0 \tag{2.3}$$

And the closed loop transfer function is,

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{600\alpha}{s^4 + 18s^3 + 95s^2 + 150s + 600\alpha\beta} \tag{2.4}$$

When analyzing the system with a nonlinearity in the forward path, β will be equal to unity. The converse is true when analyzing with the nonlinearity in the feedback path. α will then be equal to unity.

Many types of single-valued nonlinearities could be considered here. Two types are used in this example and in examples that will follow mainly for simplicity sake. These two types are saturation and dead zone. For this example the saturation element limits at 10.0. as shown in Figure (2.3a). Figure (2.3b) shows the dead zone element where the dead zone ends at 10.0. The assumed magnitude of Θ_{in} is 20.0. It could be any magnitude; however it must be specified for the analysis because the frequency response is a function of the magnitude of Θ_{in}. Because the gain of the nonlinear element is a function of the signal going into it, what is desired is a relationship between the gain of the nonlinear element and the input signal. However, it is hard to describe the signal going into the nonlinear element as a function of the nonlinear gain. It is easier to

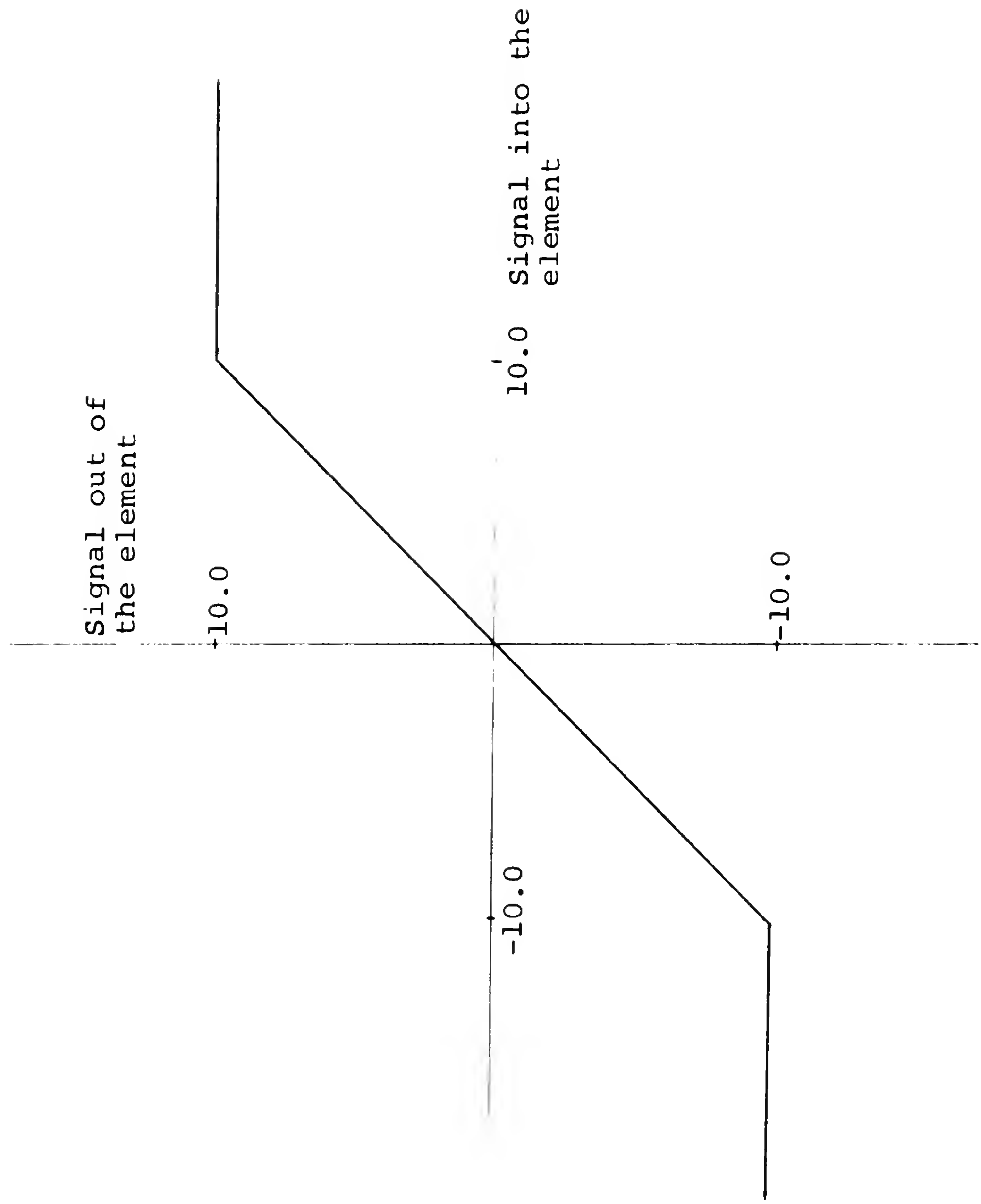

Figure 2.3a Saturation element of example in chapter 2

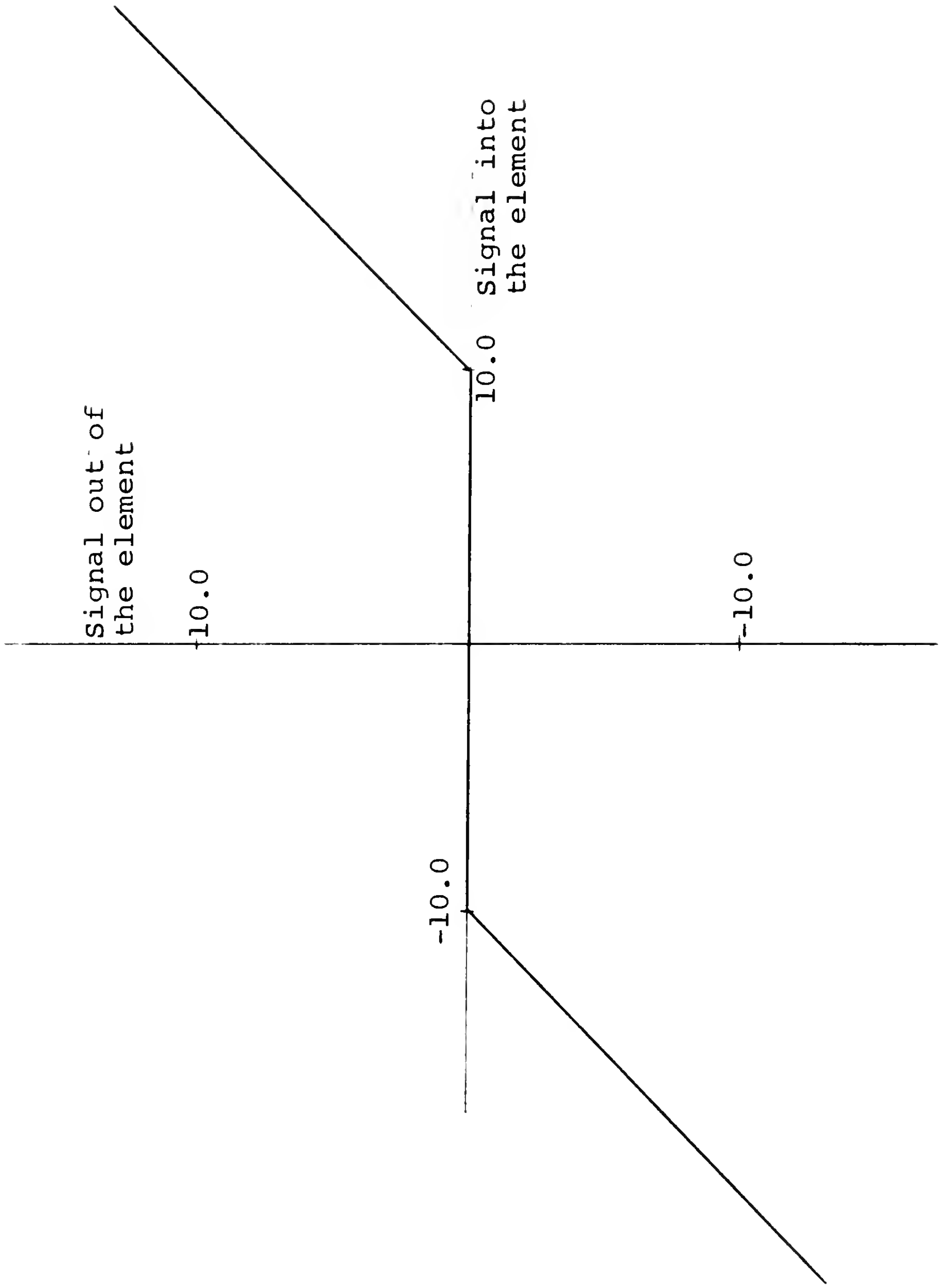

Figure 2.3b Dead zone element in chapter 2

develop a relationship between the gain of the nonlinear element and $\frac{\Theta_{nl}}{\Theta_{in}}$, where Θ_{nl} is the signal going into the nonlinear element and Θ_{in} is the input into the system. For this example, $\Theta_{nl} = \Theta_e$ for the case of the nonlinearity in the forward path, and $\Theta_{nl} = \Theta_{out}$ for the nonlinearity in the main feedback path. This may be seen in figure (2.2).

For the case of the forward path nonlinearity, the gain, α, may be described as,

$$\alpha = \frac{\Theta_b}{\Theta_e} \tag{2.5}$$

Where Θ_b is the signal out of the nonlinear element.

Dividing both the numerator and the denominator of the right-hand side of equation (2.5) by Θ_{in},

$$\alpha = \frac{\Theta_b/\Theta_{in}}{\Theta_e/\Theta_{in}} \tag{2.6}$$

Referring to figure (2.3) for the saturation element, $\Theta_b = 10$ when Θ_e is greater than 10. When $\Theta_e \leq 10$, $\Theta_b = \Theta_e$ and α is equal to unity.

Substituting $\Theta_{in} = 20$ into equation (2.6) yields,

$$\alpha = \frac{\Theta_b/20}{\Theta_e/\Theta_{in}} \tag{2.7}$$

Two cases now exist. When $\Theta_e \leq 10$, $\Theta_e = \Theta_b$ and

$$\alpha = \frac{\Theta_e/20}{\Theta_e/20} = 1.0 \qquad \Theta_e \leq 10 \qquad (2.8a)$$

When $\Theta_e > 10$, $\Theta_b = 10$ and

$$\alpha = \frac{.5}{\Theta_e/\Theta_{in}} \qquad \Theta_e \geqslant 10 \qquad (2.8b)$$

From equations (2.8a) and (2.8b), a table may be drawn up giving the gain of the saturation element as a function of $\frac{\Theta_e}{\Theta_{in}}$. This table would be the same for the saturation element in the main feedback path, by replacing Θ_e by Θ_{out}, Θ_b by Θ_a and α by β. Equations (2.8a) and (2.8b) become,

$$\beta = \frac{\Theta_{out}/20}{\Theta_{out}/\Theta_{in}} = 1.0 \qquad \Theta_{out} < 10.0 \qquad (2.9a)$$

$$\beta = \frac{.5}{\Theta_{out}/\Theta_{in}} \qquad \Theta_{out} \geq 10.0 \qquad (2.9b)$$

This table for the saturation element is table (II.1a). To use it, enter the left-hand column with $\frac{\Theta_e}{\Theta_{in}}$ or $\frac{\Theta_{out}}{\Theta_{in}}$ and read the corresponding value of α or β respectively.

In like manner the dead zone device can be dealt with. Referring once again to figure (2.2)

$$\alpha = \frac{\Theta_b}{\Theta_e} \qquad (2.5)$$

Dividing again by Θ_{in} and substituting,

$$\alpha = \frac{\Theta_b/\Theta_{in}}{\Theta_e/\Theta_{in}} = \frac{\Theta_b/20}{\Theta_e/\Theta_{in}} \qquad (2.6),(2.7)$$

Looking at figure (2.3b), it is seen that for all input signals into the dead zone element, in this case Θ_e, less than 10.0, the output, Θ_b, is zero. Therefore,

$$\alpha = \frac{0.0/20.}{\Theta_e/\Theta_{in}} = 0.0 \qquad \Theta_e \leq 10.0 \qquad (2.10a)$$

And for Θ_e greater than 10.0,

$$\alpha = \frac{\Theta_b/20}{\Theta_e/\Theta_{in}} \qquad \Theta_e \geq 10.0 \qquad (2.10b)$$

For the dead zone in the main feedback path, substitution once again obtains,

$$\beta = \frac{0.0/20.}{\Theta_{out}/\Theta_{in}} = 0.0 \qquad \Theta_{out} \leq 10.0 \qquad (2.11a)$$

$$\beta = \frac{\Theta_a/20.}{\Theta_{out}/\Theta_{in}} \qquad \Theta_{out} > 10.0 \qquad (2.11b)$$

From equations (2.11a) and (2.11b) and figure (2.3b),

$\frac{\Theta_e}{\Theta_{in}}$ or $\frac{\Theta_{out}}{\Theta_{in}}$	Saturation Element α or β	Dead Zone Element α or β
0.00	1.000	0.000
0.25	1.000	0.000
0.50	1.000	0.000
0.75	0.670	0.333
1.00	0.500	0.500
1.25	0.400	0.600
1.50	0.333	0.666
1.75	0.28	0.715
2.00	0.25	0.750
2.25	0.22	0.778
2.50	0.20	0.800
2.75	0.18	0.817
3.00	0.17	0.834
3.25	0.15	0.846
3.50	0.14	0.857
3.75	0.13	0.868
4.00	0.125	0.875
4.25	0.118	0.880
4.50	0.112	0.887
4.75	0.105	0.894
5.00	0.100	0.900
5.25	0.095	0.905
5.50	0.091	0.910
5.75	0.087	0.914
6.00	0.083	0.917
6.25	0.080	0.920
6.50	0.077	0.923
675	0.074	0.926
7.00	0.071	0.929
7.50	0.067	0.932
8.00	0.063	0.938
8.50	0.059	0.941
9.00	0.055	0.945
9.50	0.053	0.948
10.00	0.050	0.950

Table II a & b a.) α and β as Functions of $\frac{\Theta_e}{\Theta_{in}}$ for the Saturation Element b.) α and β as Functions of $\frac{\Theta_{out}}{\Theta_{in}}$ for the Dead Zone Element a table similar to table (II.1a) can be drawn up for the dead zone element. This table is table (II.1b). Once again enter with either $\frac{\Theta_e}{\Theta_{in}}$ or $\frac{\Theta_{out}}{\Theta_{in}}$ and α or β can be read off respectively.

The next step in obtaining the frequency response is to find a relationship between the nonlinear gain, α or β, and the frequency. A relationship between the nonlinear gain and the signal into the element was just shown. Using the PARAMS program, a relationship between α or β and frequency can be found. The PARAM-5 part of the program does this. Given a transfer function, it will draw a plot of gain, α or β, versus the magnitude of the transfer function with constant frequency lines drawn.

The case of the nonlinearity in the forward path will be considered first. In this case $\beta = 1.0$ and $\Theta_a = \Theta_{out}$. Looking at figure (2.2),

$$\Theta_e = \Theta_{in} - \Theta_{out} \tag{2.12}$$

Divide both sides of equation (2.12) by Θ_{in},

$$\frac{\Theta_e}{\Theta_{in}} = 1 - \frac{\Theta_{out}}{\Theta_{in}} \tag{2.13}$$

$\frac{\Theta_{out}}{\Theta_{in}}$ has already been computed as equation (2.4)

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{600\alpha}{s^4 + 18s^3 + 95s^2 + 150s + 600\alpha\beta} \tag{2.4}$$

Substituting equation (2.4) into equation (2.13) and letting $\beta = 1.0$.

$$\frac{\Theta_e}{\Theta_{in}} = \frac{s^4 + 18s^3 + 95s^2 + 150s}{s^4 + 18s^3 + 95s^2 + 150s + 600\alpha} \qquad (2.14)$$

Using equation (2.14) and letting α vary between 0.0 and 1.5, a plot as described in the previous paragraph is obtained by the PARAM-5 subprogram of the PARAMS program.

This plot is figure (2.4), with α, the nonlinear gain in the forward path as the ordinate and $\frac{\Theta_e}{\Theta_{in}}$ as the abscissa. Constant ω lines, $\omega = 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8, 2.0, 2.2, 2.4, 2.6, 2.8, 3.0, 3.2$ and 3.4, were drawn by the PARAMS program. Using table (II.1a) and table (II.1b), constant Θ_{in} lines are drawn on figure (2.4) for the saturation element and for the dead zone element. The constant Θ_{in} lines are drawn using α and $\frac{\Theta_e}{\Theta_{in}}$ as the coordinates.

After drawing the constant Θ_{in} curves on figure (2.4), it is seen that these curves intersect the constant omega curves in several places. The intersection of the constant Θ_{in} curves with the constant omega curves, with the associated value of α at each intersection, determines a relationship between α, the nonlinear gain, and ω, the frequency of the overall system. With this relationship between α and ω, the total frequency response will be obtained. Figure (2.4) shows a pair of relationships, one

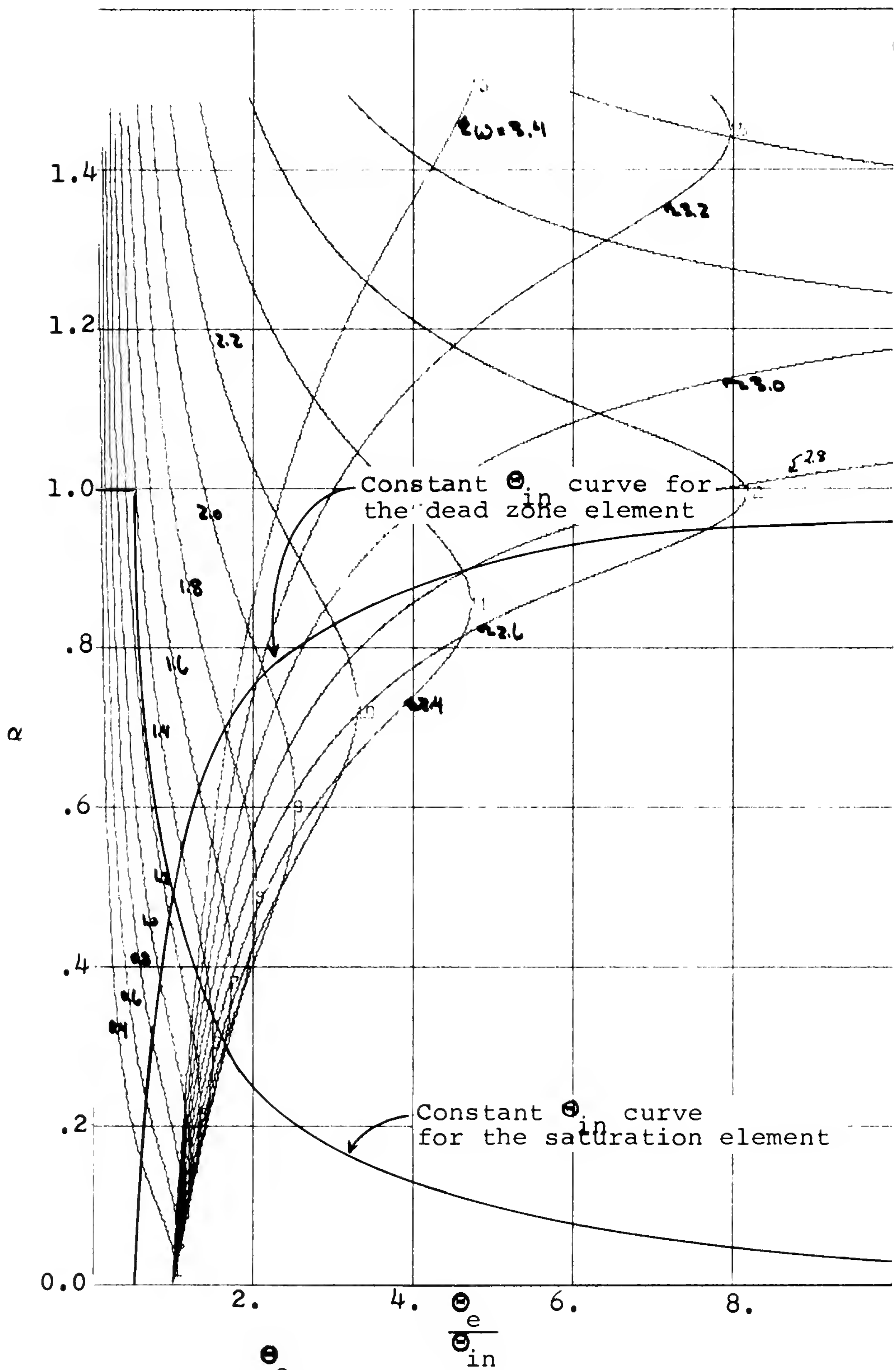

Figure 2.4 α versus $\frac{\Theta_e}{\Theta_{in}}$ magnitude for a nonlinearity in the forward path

for the saturation element and the other for the dead zone element. These two are in no way related to each other. They are both shown on the same plot so as to analyze two different cases at one time.

The final step in the analysis is obtaining the frequency response itself. Once again the PARAMS program is used. The PARAM-7 subprogram of the PARAMS program gives a frequency response plot with constant α or β lines drawn on it. Because (for this analysis) the closed loop frequency response is desired, a plot of $\frac{\Theta_{out}}{\Theta_{in}}$ magnitude (dB) versus frequency with constant α curves is obtained. This is figure (2.5) with α curves of α = 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, and 1.5 drawn on it.

Now using the relation that was found in figure (2.4) between α and ω through the intersection of the constant ω curves and the constant ω curves, these points are plotted on figure (2.5). The coordinates to transfer these points from figure (2.4) to figure (2.5) are α and ω. Where there is no exact α or ω line on figure (2.5), interpolation is used.

When looking at figure (2.5) it should be noted that the saturation frequency response shows some jump resonance around its resonance peak. This is characteristic of saturation elements and can be predicted from figure (2.4).

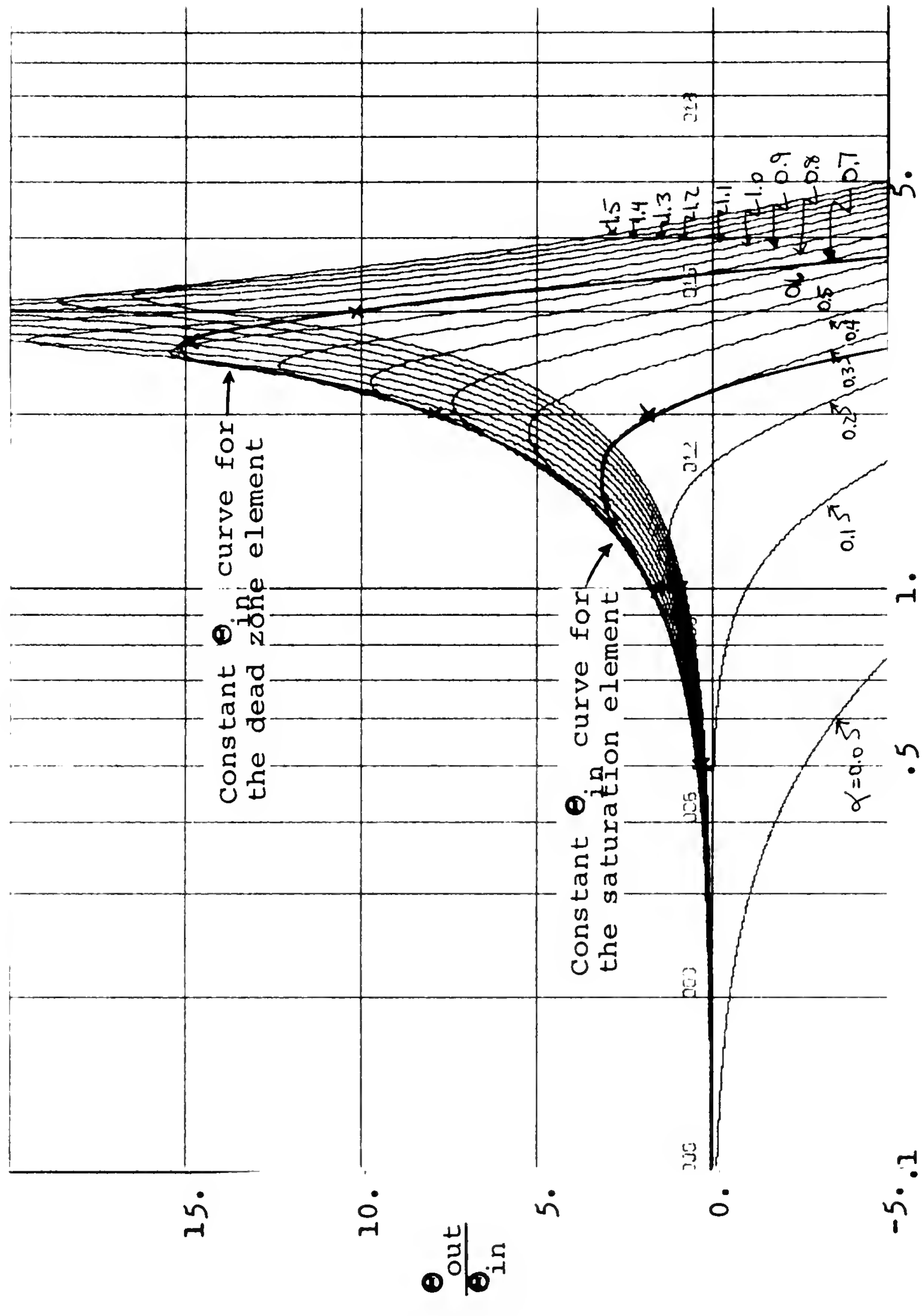

Figure 2.5 Closed loop frequency response of the servo-mechanism with a nonlinearity in the forward path

To do this, trace the constant Θ_{in} saturation curve in figure (2.4) from left to right. Note that it intersects the constant $\omega = 1.4$ curve twice giving two values of nonlinear gain, α, for $\omega = 1.4$. This is part of the jump resonance effect and will be looked at more closely in Chapter 4.

To test the results of this graphical analysis, an analog simulation of the system was performed and the actual frequency response was obtained. How this simulation was performed and how the nonlinear elements were simulated is explained in Appendix A. The analog results are plotted on figure (2.5) as 'x'. The analog simulation was close enough to justify the conclusion that the graphical analysis was accurate in predicting the frequency response. Any difference between the predicted response and actual response is due to errors in graphical construction, reading of the graph and tolerance in the simulating equipment.

The main effort of this thesis is directed to closed loop frequency response. However, the open loop frequency response may be obtained in the same manner as the closed loop response. The means of obtaining the open loop response is in the final step of the analysis. When using the PARAM-7 subroutine of the PARAMS program, instead of using the closed loop transfer function, the open loop transfer function should be used. The plot that should be obtained is the open loop magnitude (dB) versus frequency with constant α curves drawn on the plot by the program. For this first case the plot is figure (2.6). The phase curve is superimposed for more information. The next step is the same as in the first case. Using the coordinate values of α and ω found in figure (2.4), redraw the constant Θ_{in} curves on figure (2.6). The result is the open loop frequency response for a system with either saturation or dead zone in the forward path.

The next case in this example is a nonlinear element in the main feedback path. In this case $\alpha = 1.0$ and Θ_{out} is the signal into the nonlinear element as shown in figure (2.2). Equations (2.9a), (2.9b), (2.11a), and (2.11b) have been developed for the nonlinear element in the main feedback path and the results tabulated in table (2.1a) for a saturation element and table (2.1b) for a dead zone element.

Using the PARAM-5 subprogram of the PARAMS program, a plot of β, the nonlinear gain in the main feedback path, versus the magnitude of $\frac{\Theta_{out}}{\Theta_{in}}$ is obtained in the same manner as for figure (2.4). This plot is figure (2.7) and shows β between 0.0 and 1.5 as the ordinate and $\frac{\Theta_{out}}{\Theta_{in}}$ as the abscissa with constant ω curves of ω = 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8, 2.0, 2.2, 2.4, 2.6, 3.0, 3.2, and 3.4 drawn by the program on the plot.

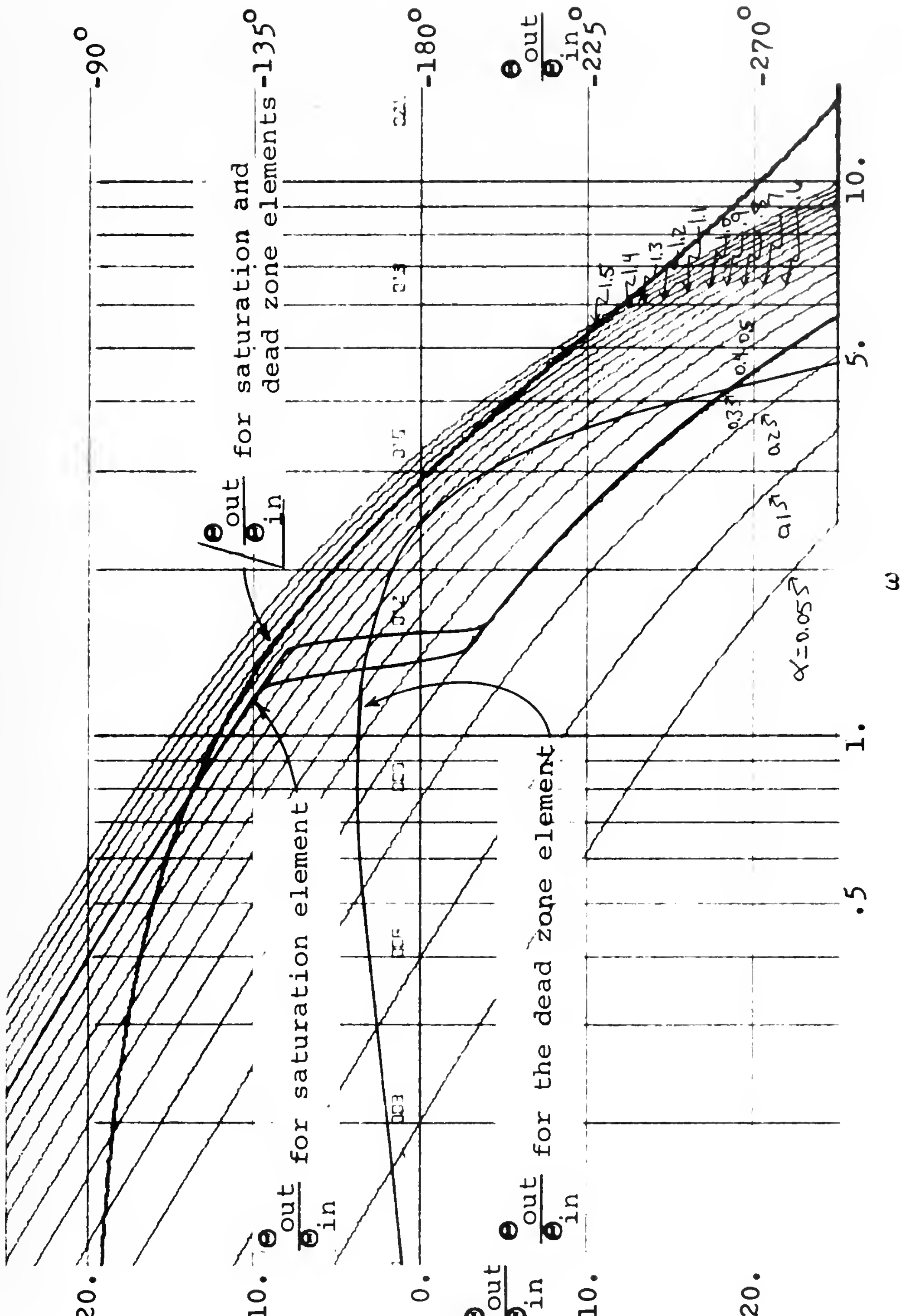

Figure 2.6 Open loop frequency response for a saturation and dead zone elements in the forward path

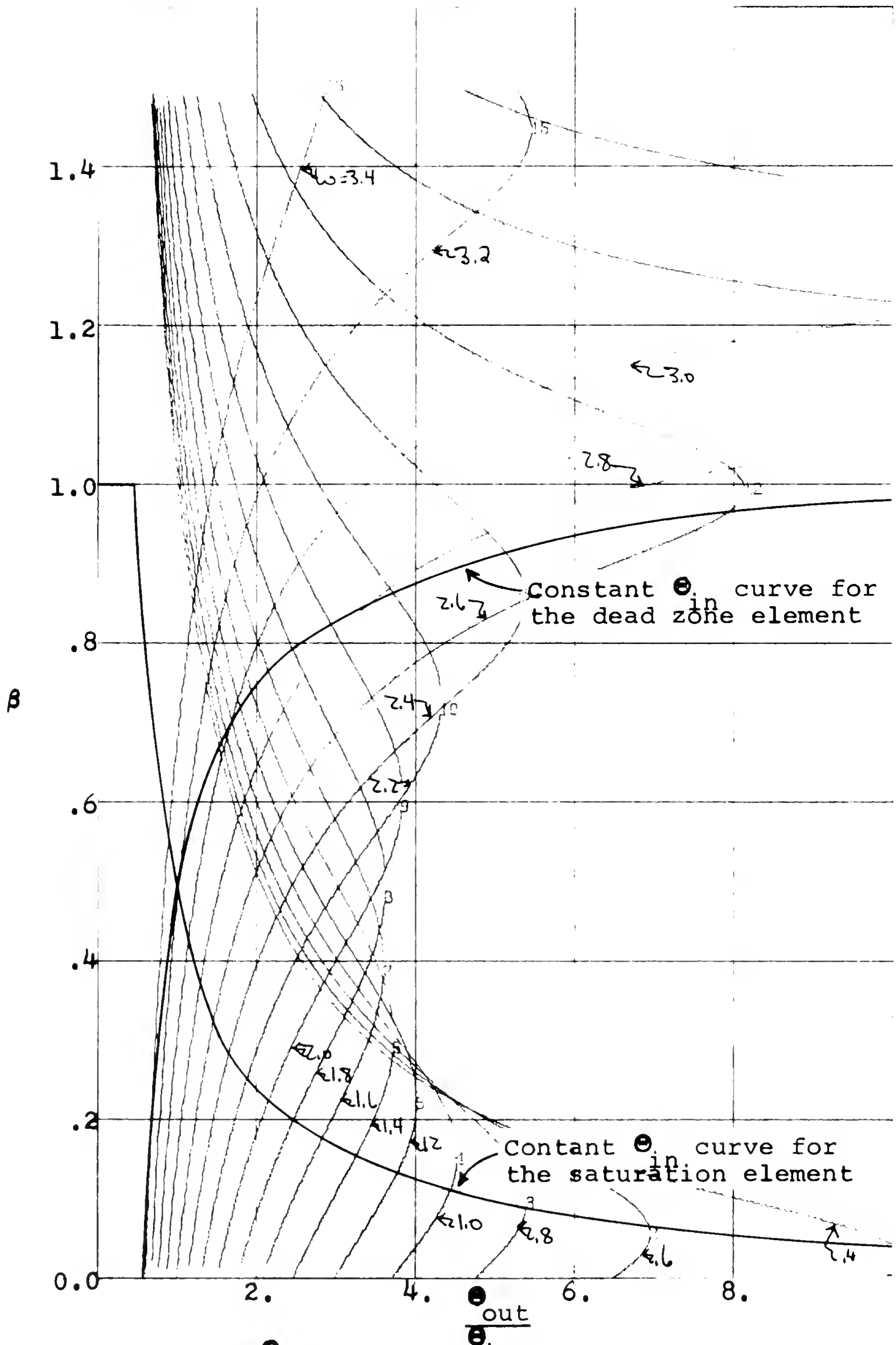

Figure 2.7 β versus $\frac{\Theta_{out}}{\Theta_{in}}$ magnitude for a nonlinearity in the feedback path

The next step in the analysis is the same as in the first case. Using the values in table (II-1a) and table (II-1b), constant Θ_{in} curves for saturation and dead zone are drawn on figure (2.7). The coordinates for drawing these two curves on figure (2.6) are β and $\frac{\Theta_{out}}{\Theta_{in}}$. The intersection in figure (2.6) of the constant Θ_{in} curves with the constant ω curves with an associated β for each intersection, gives the relationship between the nonlinear gain, β and ω for the system.

The final step in this analysis of the second case is the obtaining of the closed loop frequency response. As in the first case, the PARAM-7 subprogram of the PARAMS program is used. Using equation (2.4) with $\alpha = 1.0$ figure (2.8) is obtained with the magnitude of $\frac{\Theta_{out}}{\Theta_{in}}$ (dB) as the ordinate and frequency as the abscissa. Constant β lines of $\beta = 0.0$, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5 are drawn on the plot by the program.

Using the coordinate of the intersection on the constant Θ_{in} curves and the constant ω curves, that is β and ω, the constant Θ_{in} curves are drawn on figure (2.8). These curves on figure (2.8) give the closed loop frequency response of the overall system for a saturation element in the main feedback path and for a dead zone element. Note that there is no jump resonance in either response. This is as predicted by looking at figure (2.7) and observing that neither constant Θ_{in} curves intersects a constant omega

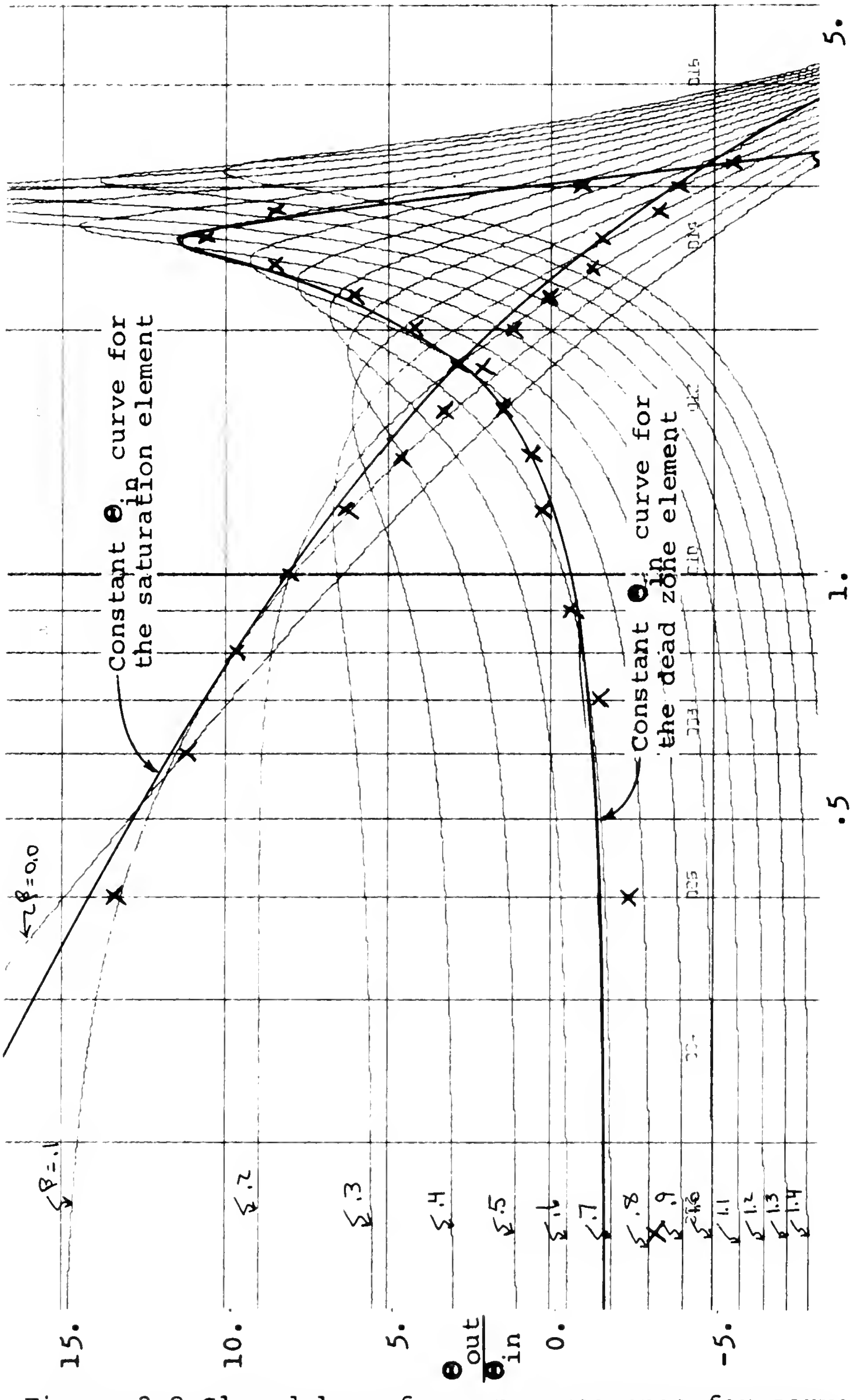

Figure 2.8 Closed loop frequency response for servo-mechanism with a nonlinearity in the feedback path

curve more than once. This could have occurred in both cases, saturation and dead zone, if the constant Θ_{in} curves had been shifted sufficiently to the right in figure (2.7). The shifting to the right of the constant Θ_{in} saturation curve corresponds to raising the value at which the signal saturates. The shifting of the constant Θ_{in} dead zone curve to the right corresponds to increasing the area of dead zone.

The results in this second case, a nonlinearity in the main feedback path, were also checked by analog simulation. The simulation results are plotted with 'x' and correspond enough to verify that the graphical analysis is correct.

C. STEPS IN THE ANALYSIS

The steps in the analysis are listed below for ease in following the process. The use of the PARAMS program makes this analysis realizable because of its speed and ease of calculation. At the present time, the PARAMS program is limited to a 49th order system.

1. Draw the block diagram of the system, with the nonlinear element as a separate block. In order to use this technique, the nonlinearity must be a single-valued type.

2. Write the transfer function of the closed loop system. Also write the transfer function of the signal preceding the nonlinearity with respect to the input signal

into the system. In some cases these two transfer functions may be the same as in the case in this chapter where the nonlinearity was in the main feedback path.

3. Using the PARAMS program, or its equivalent, obtain a plot of the nonlinear gain versus the magnitude transfer function of the signal into the nonlinearity. This plot should have drawn on it constant ω curves for the frequencies of interest.

4. Obtain a relationship between the nonlinear gain and magnitude of the transfer function of the signal into the nonlinearity. This can be done several ways, one of which is described in this chapter.

5. Plot this relationship found in step 4 onto the plot drawn in step 3 using the nonlinear gain and transfer function magnitude as the coordinates. This is the constant Θ_{in} curve.

6. The intersection of the constant Θ_{in} curve and the constant ω curves gives an associated value of nonlinear gain for each intersection. These intersections give the relationship between the nonlinear gain and frequency. In some cases there may be more than one associated value of gain for a certain frequency. This can be ascribed to a jump resonance effect.

7. Again using the PARAMS program or its equivalent, obtain a standard frequency response plot, either open or closed loop, with constant nonlinear gain curves drawn on

it. The values of nonlinear gain curves must correspond to the range of nonlinear gain found in step 6.

8. Using the values of nonlinear gain and frequency found in step 6 as coordinates, replot the constant Θ_{in} curve on the plot found in step 7. This gives the closed or open loop frequency response - whatever the case may be - of a system with a nonlinear element to a constant magnitude input signal.

D. FURTHER INVESTIGATION OF THE JUMP RESONANCE EFFECT

The frequency response for the system with saturation in the forward path, figure (2.5), shows a jump resonance around $\omega = 1.4$. This jump resonance is to be expected with a nonlinear device. In Chapter 4 this effect will be shown for a dead zone element. To illustrate this jump resonance more clearly, another example will be shown here. The example is a second order feedback system which is used as an example in a nonlinear control textbook.[2] Its block diagram is figure (2.9).

By letting α represent the variable gain of the saturation element, the closed loop transfer function may be written as,

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{200\alpha}{s^2 + s + 200\alpha} \qquad (2.15)$$

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[2] Thaler, G. J. and Pastel, M.P., Analysis and Design of Nonlinear Feedback Control Systems, p. 198, McGraw-Hill, 1962.

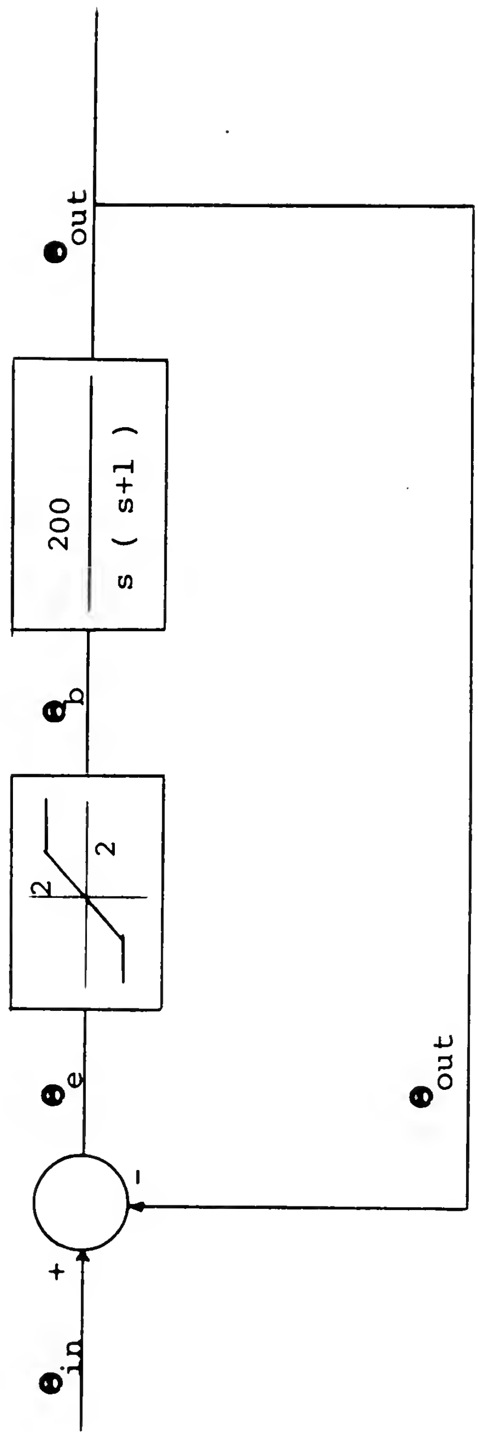

Figure 2.9 Block diagram of a servo with saturation

What is desired is the frequency response with a constant input magnitude. Θ_{in} is chosen to be $\Theta_{in} = 2.0$.

Following the steps in the analysis technique, the transfer function of the signal preceding the nonlinear element with respect to Θ_{in}, that is $\frac{\Theta_e}{\Theta_{in}}$, is derived

$$\Theta_e = \Theta_{in} - \Theta_{out} \tag{2.16}$$

Dividing by Θ_{in},

$$\frac{\Theta_e}{\Theta_{in}} = 1 - \frac{\Theta_{out}}{\Theta_{in}} \tag{2.17}$$

And substituting equation (2.15) into equation (2.17),

$$\frac{\Theta_e}{\Theta_{in}} = \frac{s^2 + s}{s^2 + s + 200\alpha} \tag{2.18}$$

By using the PARAMS-5 subprogram of the PARAMS and equation (2.18), a plot of α versus $\frac{\Theta_e}{\Theta_{in}}$ with constant ω curves is obtained. This plot is figure (2.10).

Next the relation between $\frac{\Theta_e}{\Theta_{in}}$ and α is found.

Referring to figure (2.9)

$$\alpha = \frac{\Theta_b}{\Theta_e}$$

Dividing both the numerator and denominator of the right-hand side of equation (2.19) by Θ_{in},

$$\alpha = \frac{\Theta_b/\Theta_{in}}{\Theta_e/\Theta_{in}} \tag{2.20}$$

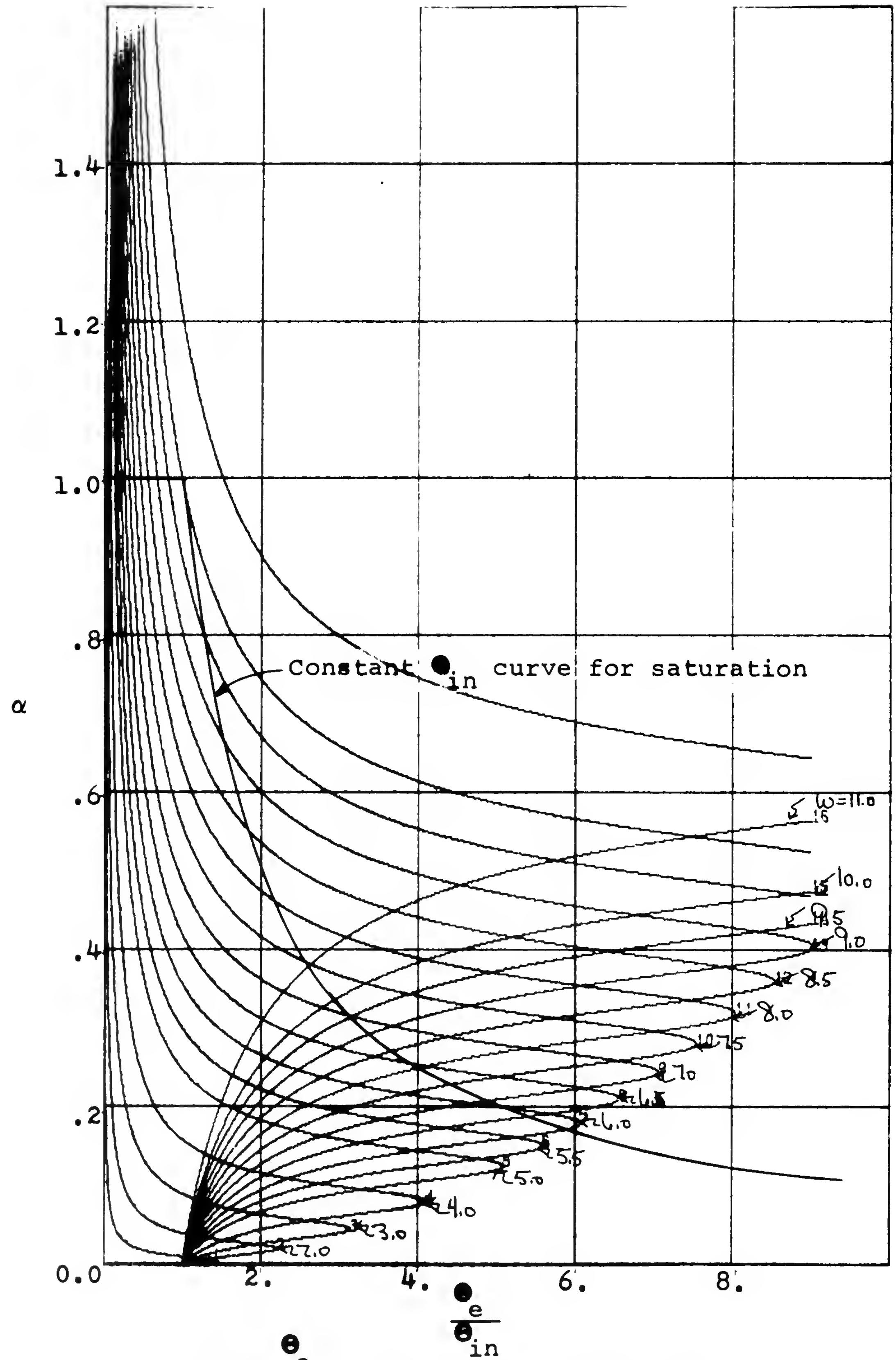

Figure 2.10 α versus $\frac{\Theta_e}{\Theta_{in}}$ magnitude for servo with saturation

and when $\Theta_e \leqslant 2.0$, $\alpha = 1$ and when $\Theta_b > 2.0$, $\frac{\alpha}{\Theta_b} = 2.0$ as the element goes into saturation. Two cases exist for $\Theta_{in}=2.0$.

$$\alpha = 1.0 \qquad \frac{\Theta_e}{\Theta_{in}} \leq 1.0 \tag{2.21a}$$

$$\alpha = \frac{1.0}{\Theta_e/\Theta_{in}} \qquad \frac{\Theta_e}{\Theta_{in}} \geq 1.0 \tag{2.21b}$$

With equations (2.21a) and (2.21b), a table is constructed giving α for the corresponding values of $\frac{\Theta_e}{\Theta_{in}}$. This table is table (II.2) where $0.0 \leq \frac{\Theta_e}{\Theta_{in}} \leq 10.0$.

Using the values in the table, the constant Θ_{in} curve is plotted on figure (2.10). The intersection of this constant Θ_{in} curve and the ω curves in figure (2.10) gives the relation between ω and α for a constant $\Theta_{in} = 2.0$.

The closed loop frequency response of the servo can now be obtained for a $\Theta_{in} = 2.0$. Using the PARAM-7 subprogram and equation (2.15), a plot of $\frac{\Theta_{out}}{\Theta_{in}}$ (dB) versus ω is obtained. This is figure (2.11) and has constant α curves of α = 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.7, 0.8, 0.9 and 1.0 plotted on it. Using the coordinates of α and ω for the constant Θ_{in} curve in figure (2.10), the constant Θ_{in} curve is replotted onto figure (2.11). As may be seen, there is a large jump resonance effect between $\omega = 6.0$ and $\omega = 10.0$.

The jump resonance effect can also be seen in the phase curve of this example. The technique is similar to that of drawing the magnitude frequency response. Using

$\frac{\Theta_e}{\Theta_{in}}$	α
0.0	1.0
0.5	1.0
1.0	1.0
1.5	0.667
2.0	0.500
2.5	0.400
3.0	0.333
3.5	0.286
4.0	0.250
4.5	0.222
5.0	0.200
5.5	0.182
6.0	0.167
6.5	0.154
7.0	0.143
7.5	0.133
8.0	0.125
8.5	0.118
9.0	0.111
9.5	0.105
10.0	0.100

Table II.2 α as a Function of $\frac{\Theta_e}{\Theta_{in}}$ for the Saturation Element

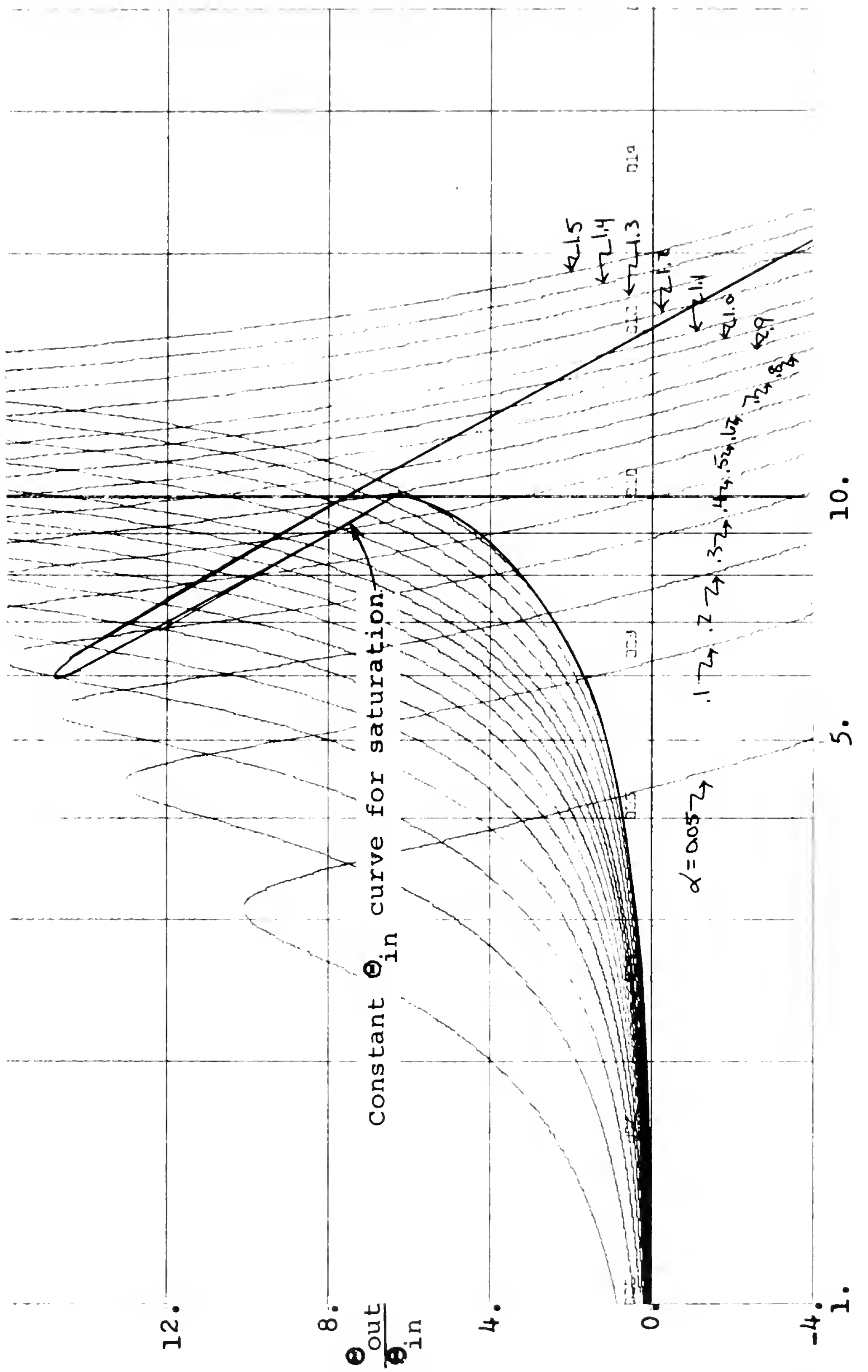

Figure 2.11 Closed loop magnitude frequency response for a a servo with saturation

the PARAM-7 subprogram again, a plot of the phase of $\frac{\Theta_{out}}{\Theta_{in}}$ versus omega with the same constant α curves is drawn. This is figure (2.12). Using the coordinates of α and ω from figure (2.10), the constant Θ_{in} curve is drawn on the phase plot, figure (2.12). The jump resonance effect is seen again, though not as pronounced.

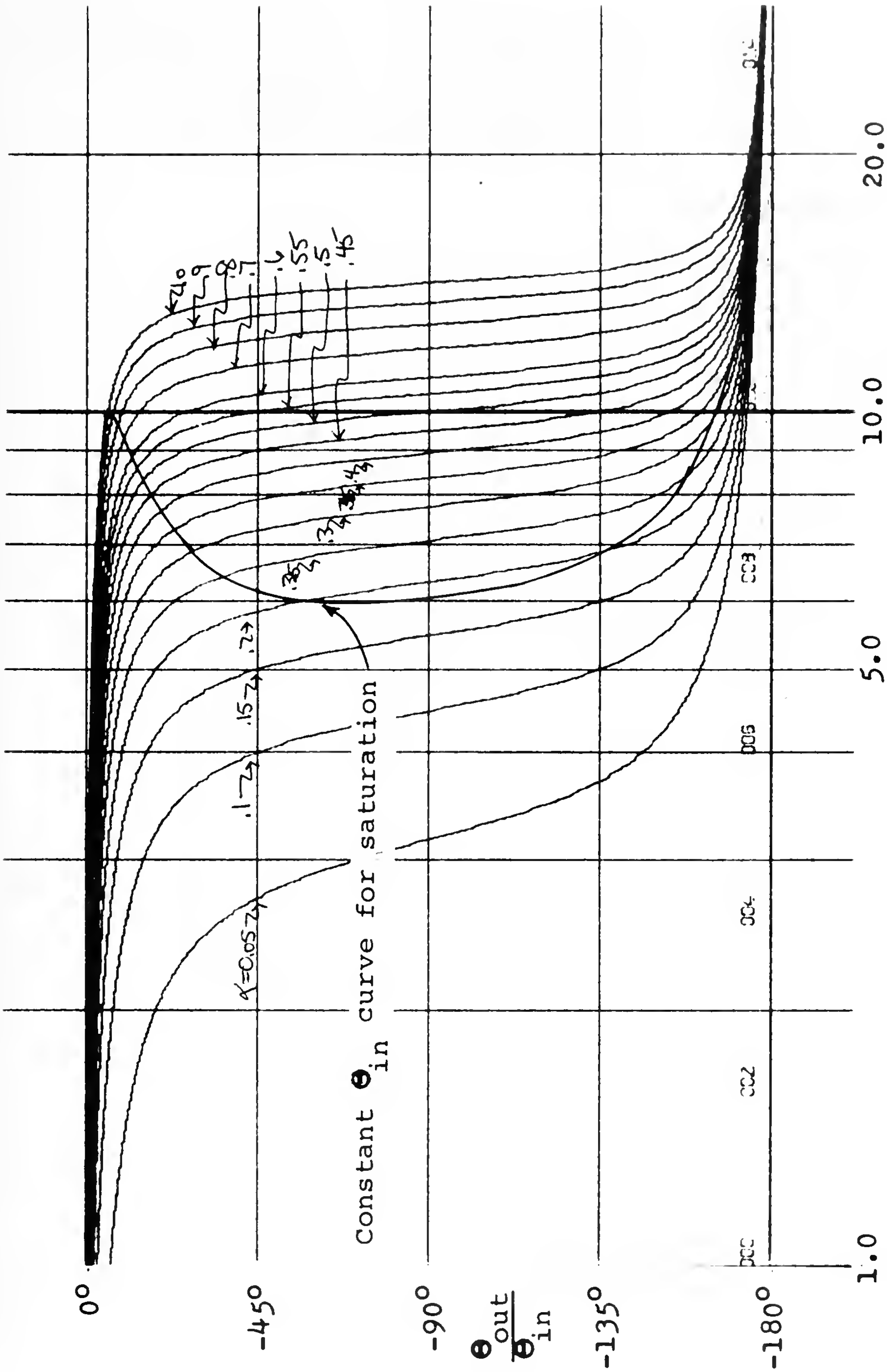

Figure 2.12 Closed loop phase frequency response for the servo with saturation

III. FURTHER EXAMPLES OF NONLINEAR FREQUENCY RESPONSE ANALYSIS

A. INTRODUCTION

In this chapter, six examples are worked in order to further verify the validity of the analysis technique. Saturation, dead zone, an ideal relay, and coulomb frction plus stiction are the nonlinearities used in the examples. These nonlinearities are placed in various positions in the systems including the forward path and the minor feedback path. The first four examples were checked by analog simulation. Also in all six examples Θ_{in} was assumed to be of magnitude $\Theta_{in} = 10.0$.

B. FIRST EXAMPLE

The first example to be worked is a third order system with a saturation element in the forward path. Its block diagram is figure (3.1) where α represents the variable nonlinear gain of the saturation element. From figure (3.1),

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{9\alpha}{s^3 + s^2 + 16.25s + 9\alpha} \qquad (3.1)$$

The saturation element has unity gain for Θ_e signals less than 50.0. Signals into the element greater than 50.0

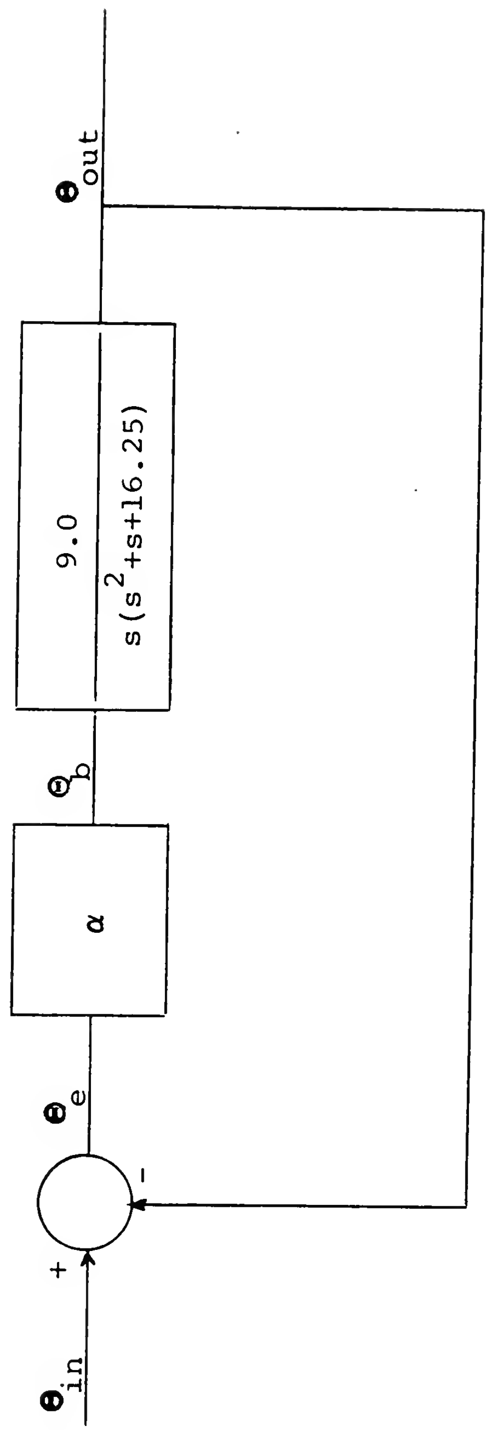

Figure 3.1 Block diagram of example #1 where α represents the variable gain of the saturation element

are limited so that the output of the element is 50.0. The input-output diagram of the element is figure (3.2). Referring to figure (3.1),

$$\alpha = \frac{\Theta_b}{\Theta_e} \tag{3.2}$$

Dividing both the numerator and the denominator of the right-hand side of equation (3.2) by Θ_{in}

$$\alpha = \frac{\Theta_b/\Theta_{in}}{\Theta_e/\Theta_{in}} \tag{3.3}$$

For a $\Theta_{in} = 10.0$, equation (3.3) can be divided into two parts.

$$\alpha = 1.0 \qquad \frac{\Theta_e}{\Theta_{in}} \leq 5.0 \tag{3.4a}$$

$$\alpha = \frac{5.0}{\Theta_e/\Theta_{in}} \qquad \frac{\Theta_e}{\Theta_{in}} \geq 5.0 \tag{3.4b}$$

Equations (3.4a) and (3.4b) define the relationship between α and $\frac{\Theta_e}{\Theta_{in}}$. It was seen in the previous chapter that this relationship is necessary to obtain the frequency response. These equations are tabulated in table (II.1), where the left-hand column is $\frac{\Theta_e}{\Theta_{in}}$ and α is the value of nonlinear gain for the particular $\frac{\Theta_e}{\Theta_{in}}$.

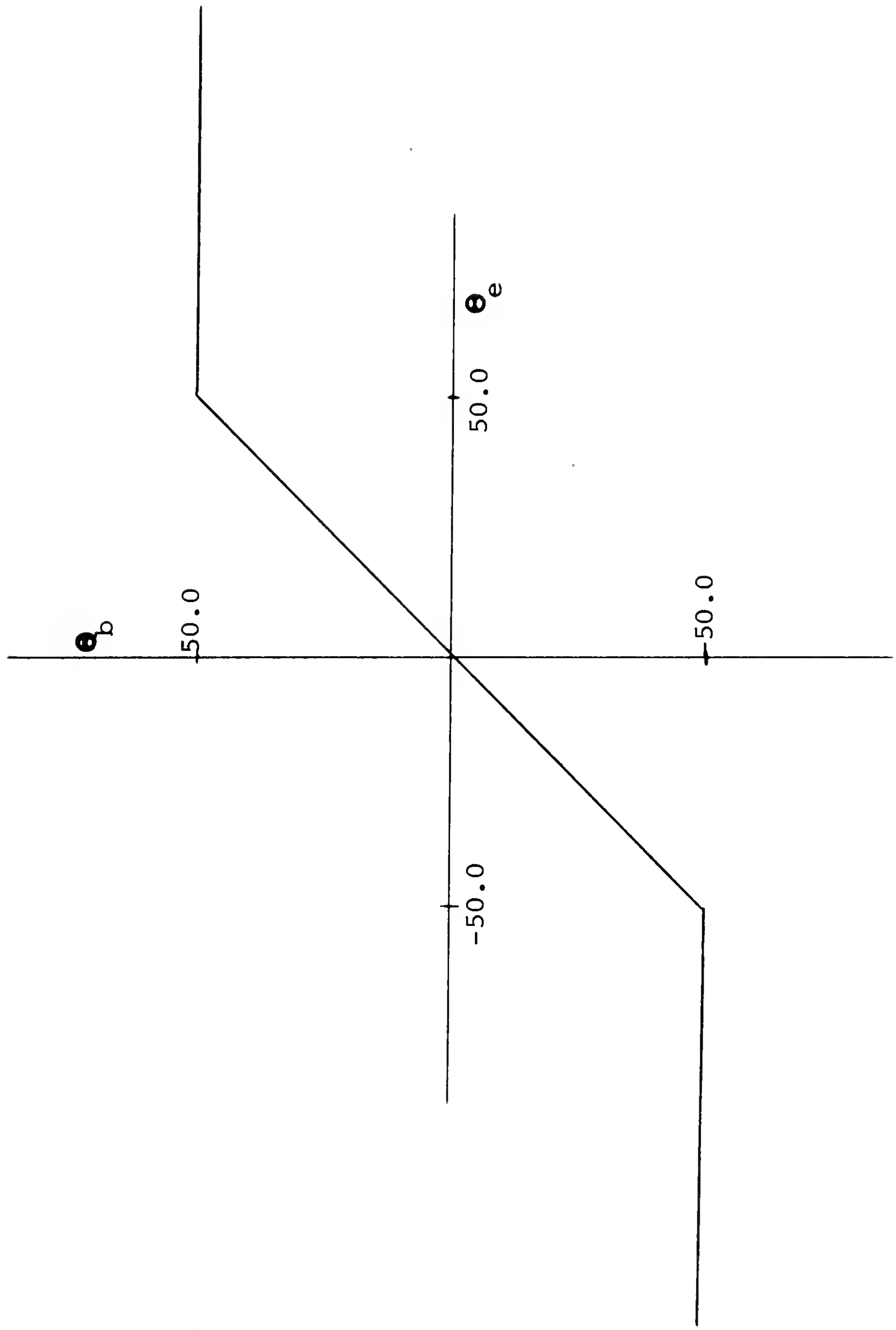

Figure 3.2 Saturation element of example #1

Having obtained the relationship between α and $\frac{\Theta_e}{\Theta_{in}}$, the relationship between α and ω is found. Referring to figure (3.1),

$$\Theta_e = \Theta_{in} - \Theta_{out}$$

$$\frac{\Theta_e}{\Theta_{in}} = \frac{\Theta_{in} - \Theta_{out}}{\Theta_{in}} = 1 - \frac{\Theta_{out}}{\Theta_{in}} \qquad (3.5)$$

Substituting equation (3.1) into equation (3.5),

$$\frac{\Theta_e}{\Theta_{in}} = \frac{s^3 + s^2 + 16.25}{s^3 + s^2 + 16.25 + 9\alpha} \qquad (3.6)$$

Using equation (3.6) and the PARAM-5 subprogram, figure (3.3) is obtained where α is the abscissa and $\frac{\Theta_e}{\Theta_{in}}$ is the ordinate. Constant ω curves are drawn on figure (3.3).

A constant Θ_{in} curve is then drawn on figure (3.3) using the coordinates of α and $\frac{\Theta_e}{\Theta_{in}}$ taken from table (III.1). The intersection of the constant Θ_{in} curve and the ω curves gives the relationship between α and ω for this analysis.

The next step is drawing the closed loop frequency response. A plot is made, using equation (3.1) and the PARAM-7 subprogram, of $\frac{\Theta_{out}}{\Theta_{in}}$ (dB) versus ω. This plot is figure (3.4) and has constant α curves on it over the range of α that is used. Using the coordinates of α and ω

$\frac{\Theta_e}{\Theta_{in}}$	α
0.0	1.0
1.0	1.0
2.0	1.0
3.0	1.0
4.0	1.0
5.0	1.0
6.0	0.833
7.0	0.715
8.0	0.625
9.0	0.555
10.0	0.500
11.0	0.454
12.0	0.416
13.0	0.385
14.0	0.357
15.0	0.333

Table III.1 α as a Function of $\frac{\Theta_e}{\Theta_{in}}$ for the Saturation Element in Example #1

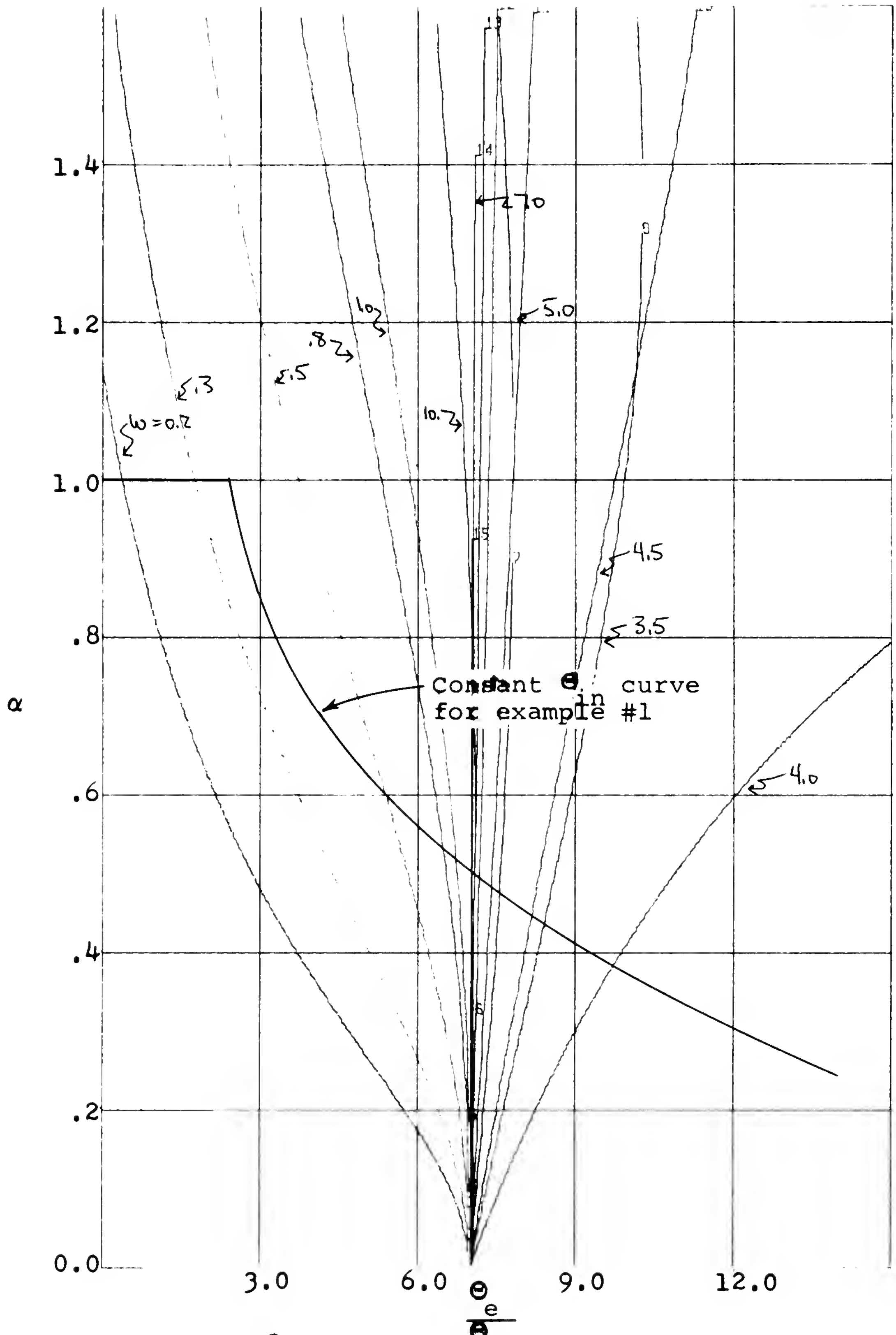

Figure 3.3 α versus $\frac{\Theta_e}{\Theta_{in}}$ magnitude for example #1

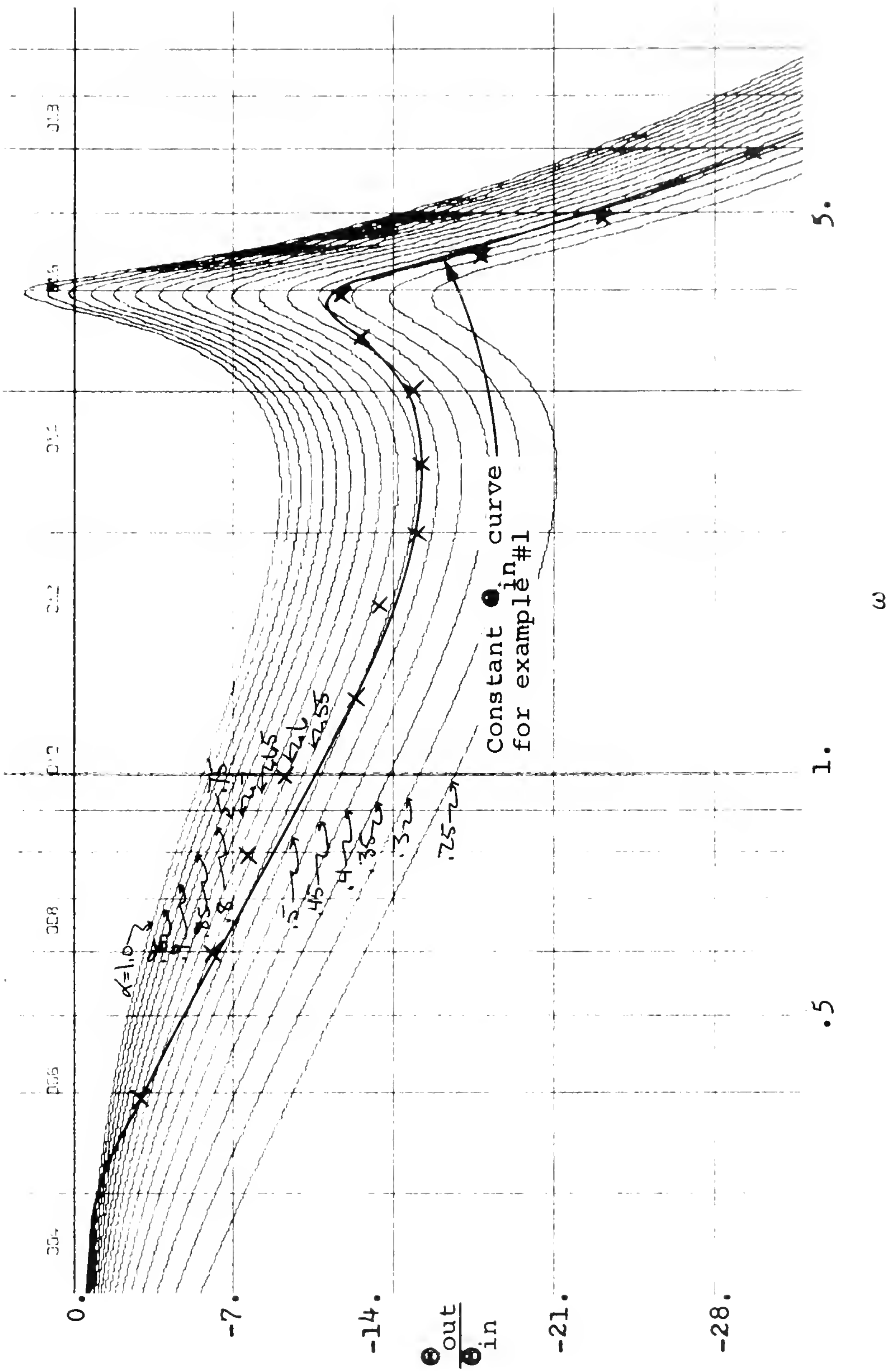

Figure 3.4 Closed loop frequency response of example #1

from figure (3.3), the constant Θ_{in} curve is redrawn on figure (3.4). This constant Θ_{in} curve on figure (3.4) is the closed loop frequency response of the first example with the saturation element in the system. The analog simulation is denoted by 'x'.

C. SECOND EXAMPLE

The second example used for analysis is a third order system with a dead zone element in the tachometer feedback path. The block diagram of the system is figure (3.5) where β represents the variable nonlinear gain of the dead zone element. From figure (3.5), the closed loop transfer function is written,

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{9}{s^3 + 16.25s + 9 + \beta s^2} \tag{3.7}$$

The dead zone element is shown in figure (3.6). For a Θ_a less than 4.0, Θ_d, the output, is zero. For a constant input of $\Theta_{in} = 10.0$, this condition exists when $\frac{\Theta_a}{\Theta_{in}}$ is less than 0.4. In equation form,

$$\beta = \frac{\Theta_d}{\Theta_a} = \frac{0.0}{\Theta_a} = 0.0 \qquad \frac{\Theta_a}{\Theta_{in}} \leq 0.4 \tag{3.7}$$

From figure (3.6), it is seen that for Θ_a greater than 4.0, $\Theta_d = 3\,(\Theta_a - 4.0)$. Thus,

$$\beta = \frac{\Theta_d}{\Theta_a} = \frac{3(\Theta_a - 4.0)}{\Theta_a} \qquad \Theta_a \geq 4.0 \tag{3.8}$$

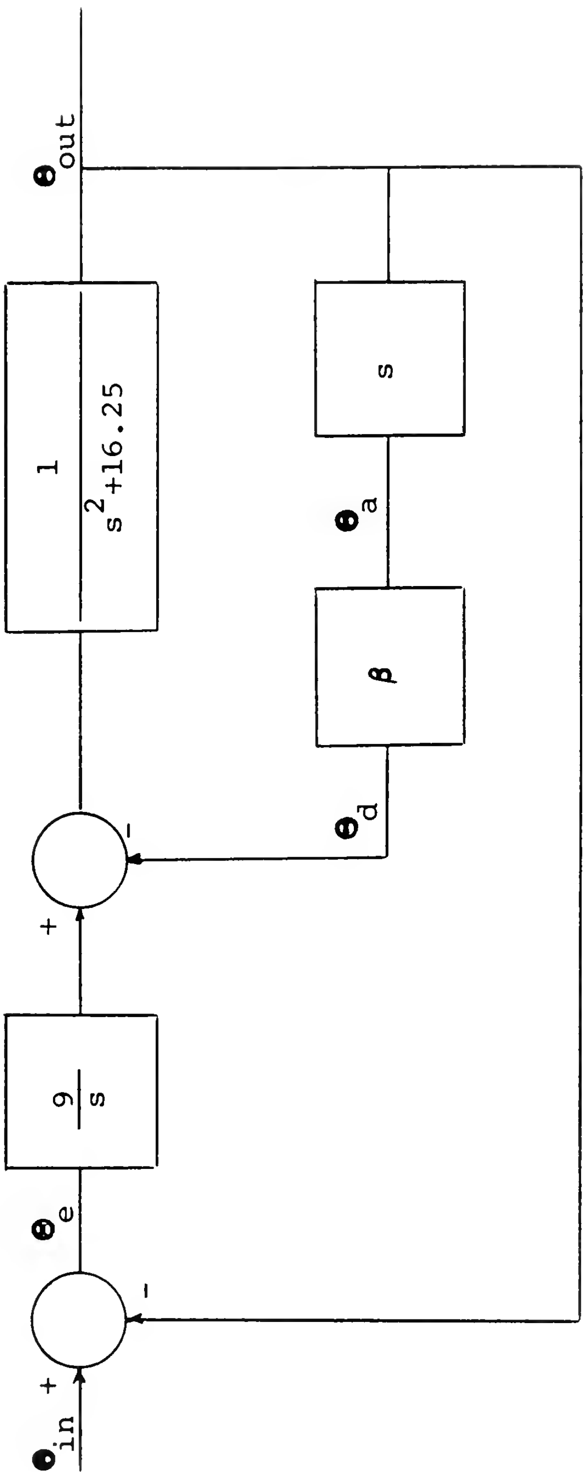

Figure 3.5 Block diagram of example #2 where β represents the variable gain of the dead zone element

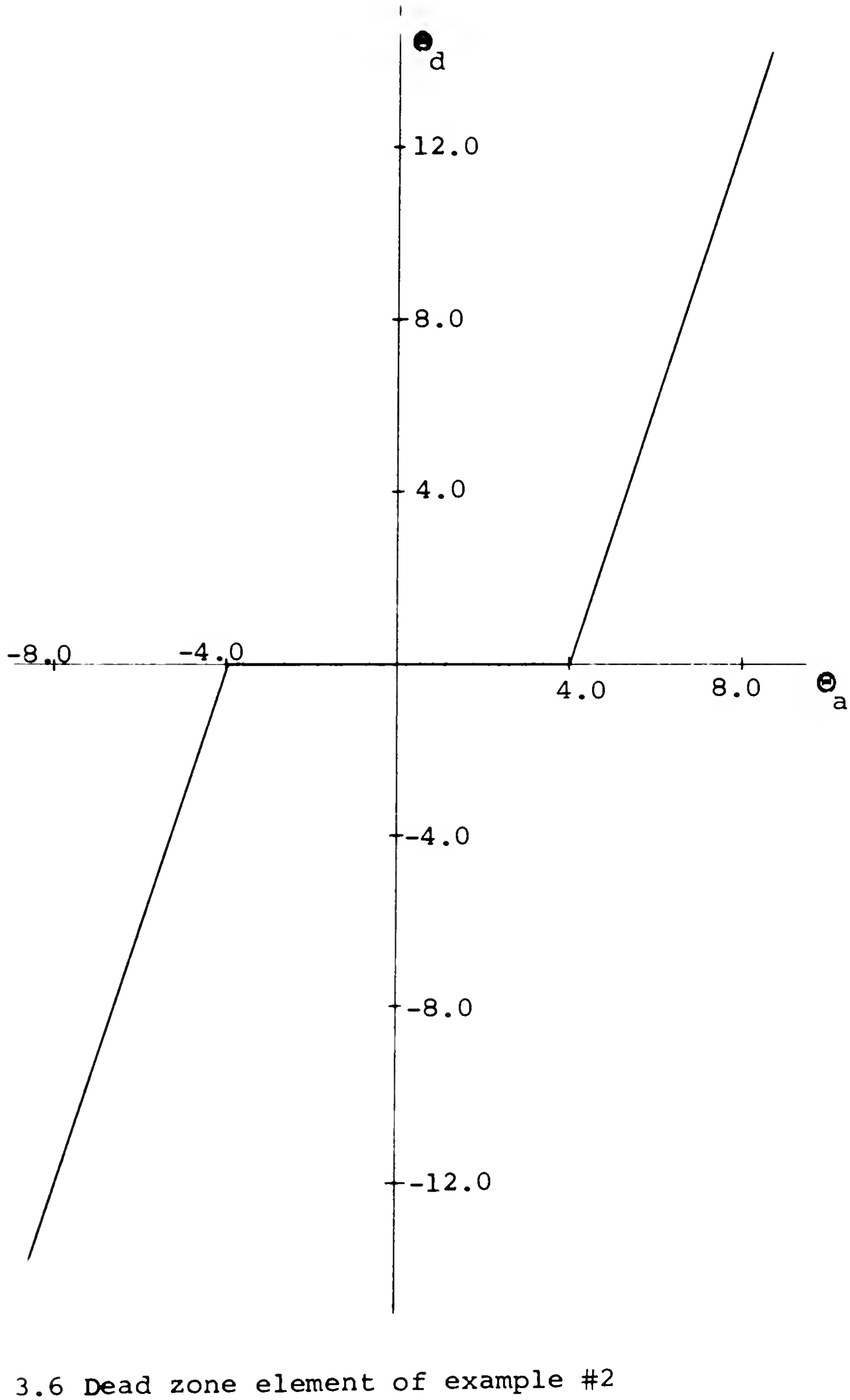

Figure 3.6 Dead zone element of example #2

Dividing the numerator and denominator of the right-hand side of equation (3.8) by Θ_{in} when $\Theta_{in} = 10.0$,

$$\beta = \frac{(3\Theta_a - 120)/\Theta_{in}}{\Theta_a/\Theta_{in}} = \frac{\frac{3\Theta_a}{\Theta_{in}} - 12.0}{\Theta_a/\Theta_{in}} \qquad \frac{\Theta_a}{\Theta_{in}} \geq 0.4 \tag{3.9}$$

Equations (3.7) and (3.9) give the relation between β and $\frac{\Theta_e}{\Theta_{in}}$. From these two equations, a table has been tabulated of β as a function $\frac{\Theta_a}{\Theta_{in}}$ over the range of $\frac{\Theta_a}{\Theta_{in}}$ desired. This table is table (III.2) where $\frac{\Theta_a}{\Theta_{in}}$ is the left-hand column and β is the value read out for the corresponding value of $\frac{\Theta_a}{\Theta_{in}}$.

Having found the relationship between β and $\frac{\Theta_a}{\Theta_{in}}$, the relationship between β and ω is now found. Referring to figure (3.5),

$$\frac{\Theta_a}{\Theta_{out}} = s \tag{3.10}$$

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{9}{s^3 + 16.25s + 9 + \beta s^2} \tag{3.7}$$

Multiplying equation (3.10) by equation (3.7),

$$\frac{\Theta_a}{\Theta_{in}} = \frac{9s}{s^3 + 16.25s + 9 + \beta s^2} \tag{3.11}$$

$\frac{\Theta_a}{\Theta_{in}}$	β
0.0	0.0
0.05	0.0
0.10	0.0
0.15	0.0
0.20	0.0
0.25	0.0
0.30	0.0
0.35	0.0
0.40	0.0
0.45	0.333
0.50	0.600
0.55	0.819
0.60	1.00
0.65	1.16
0.70	1.28
0.75	1.40
0.80	1.50
0.85	1.59
0.90	1.67
0.95	1.74
1.00	1.80

Table III.2 β as a Function of $\frac{\Theta_a}{\Theta_{in}}$ for the Dead Zone Element of Example #2

Using equation (3.11) and the PARAM-5 subprogram, a plot of β versus the magnitude of $\frac{\Theta_a}{\Theta_{in}}$ is obtained. This is figure (3.7) and has constant ω curves of $\omega = 1.0$, 2.0, 3.0, 3.5, 3.7, 3.8, 3.9, 4.0, 4.1, 4.2, 4.3, 4.4, 5.0, 6.0, 7.0 and 10.0 drawn on it.

The constant Θ_{in} curve for the dead zone element of this example has been drawn on figure (3.7). The coordinates for this curve are β and $\frac{\Theta_a}{\Theta_{in}}$, and were taken from table (III.2). The intersection of the constant Θ_{in} curve and the ω curves in figure (3.7) gives the relationship between β and ω for the system. Using this new relationship, the closed loop frequency response can now be drawn.

Equation (3.7) and the PARAM-7 subroutine are used to obtain a plot of $\frac{\Theta_{out}}{\Theta_{in}}$ versus ω with constant β curves over the range of $\beta = 0.0$, 0.1, 0.2, 0.4, 0.6, 0.8, 0.9, and 1.0. Using the coordinates of β and ω for the constant Θ_{in} curve on figure (3.7), the constant Θ_{in} curve is redrawn on figure (3.8). This constant Θ_{in} curve on figure (3.8) gives the closed loop frequency response of this second example with the dead zone element of figure (3.6) in the tachometer feedback path. The frequency response was checked by analog simulation and is denoted by 'x'.

D. THIRD EXAMPLE

The third example is a fourth order system with a dead zone element in the forward path. The block diagram is figure (3.9) where α represents the variable gain of the

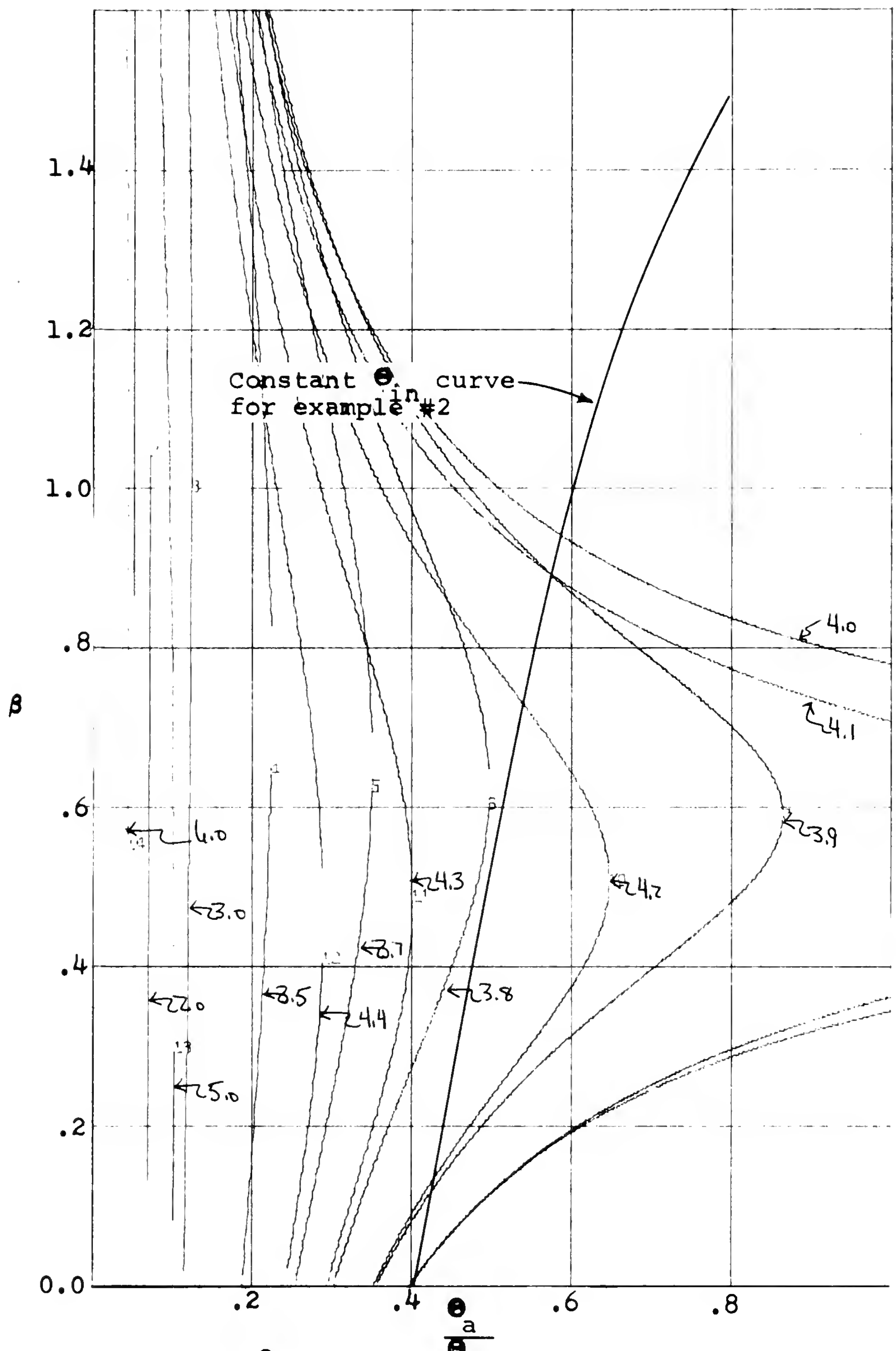

Figure 3.7 β versus $\frac{\theta_a}{\theta_{in}}$ magnitude for example #2

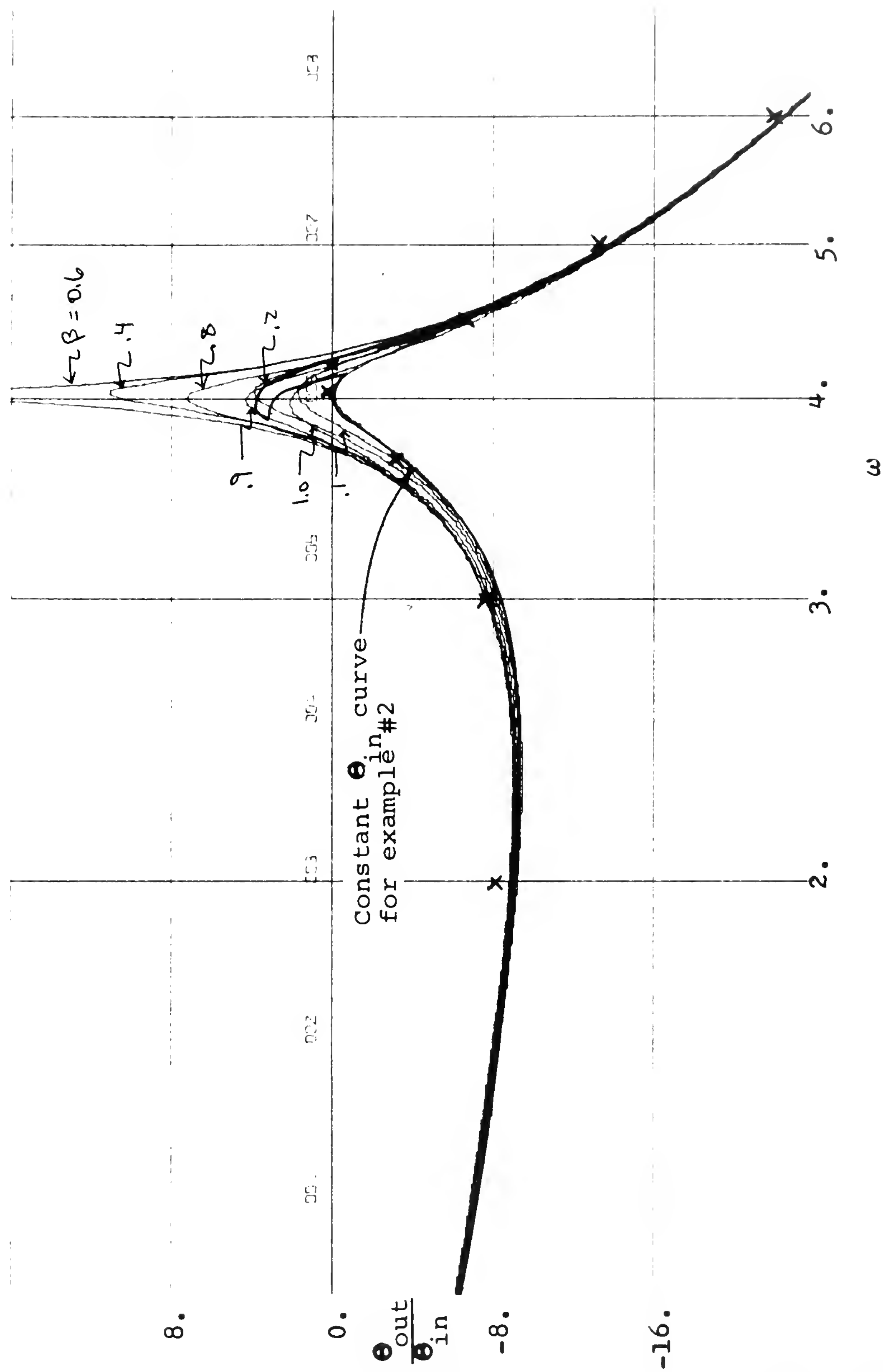

Figure 3.8 Closed loop frequency response of example #2

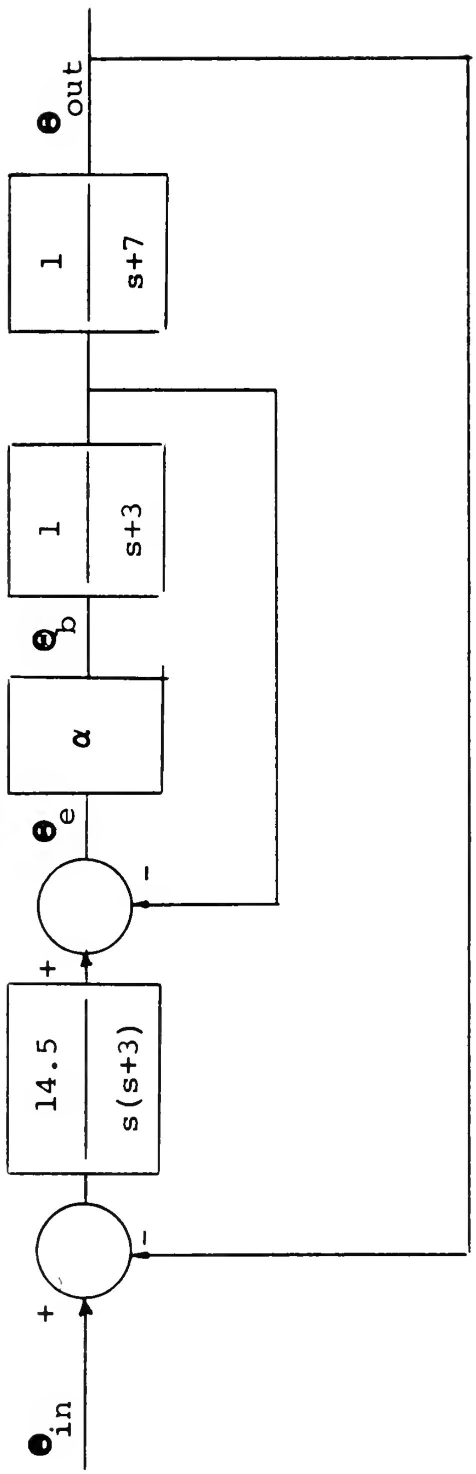

Figure 3.9 Block diagram of example #3 where α represents the variable gain of the dead zone element

dead zone element. Θ_e is the signal going into the element and Θ_b is the signal coming out of the element. From figure (3.9), the closed loop transfer function is,

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{14.5\alpha}{s^4+13s^3+51s^2+63s+(s^3+10s^2+21s+14.5)\alpha} \qquad (3.12)$$

The dead zone element is shown in figure (3.10) where the ordinate is Θ_b and the abscissa is Θ_e. The dead space extends to $\Theta_e = 50.0$. For a Θ_e greater than 50.0, the gain of the element is two. From figure (3.9),

$$\beta = \frac{\Theta_b}{\Theta_e} \qquad (3.13)$$

For $\Theta_e \leq 50.0$,

$$\beta = \frac{0.0}{\Theta_e} = 0.0 \qquad \frac{\Theta_e}{\Theta_{in}} \leq 5.0 \qquad (3.14$$

For $\Theta_e \geq 50.0$, Θ_b can be written as,

$$\Theta_b = 2(\Theta_e - 50.0) = 2\Theta_e - 100.0 \qquad (3.15)$$

Substituting equation (3.15) into equation (3.13),

$$\beta = \frac{2\Theta_e - 100.0}{\Theta_e} \qquad \Theta_e \geq 50.0 \qquad (3.16)$$

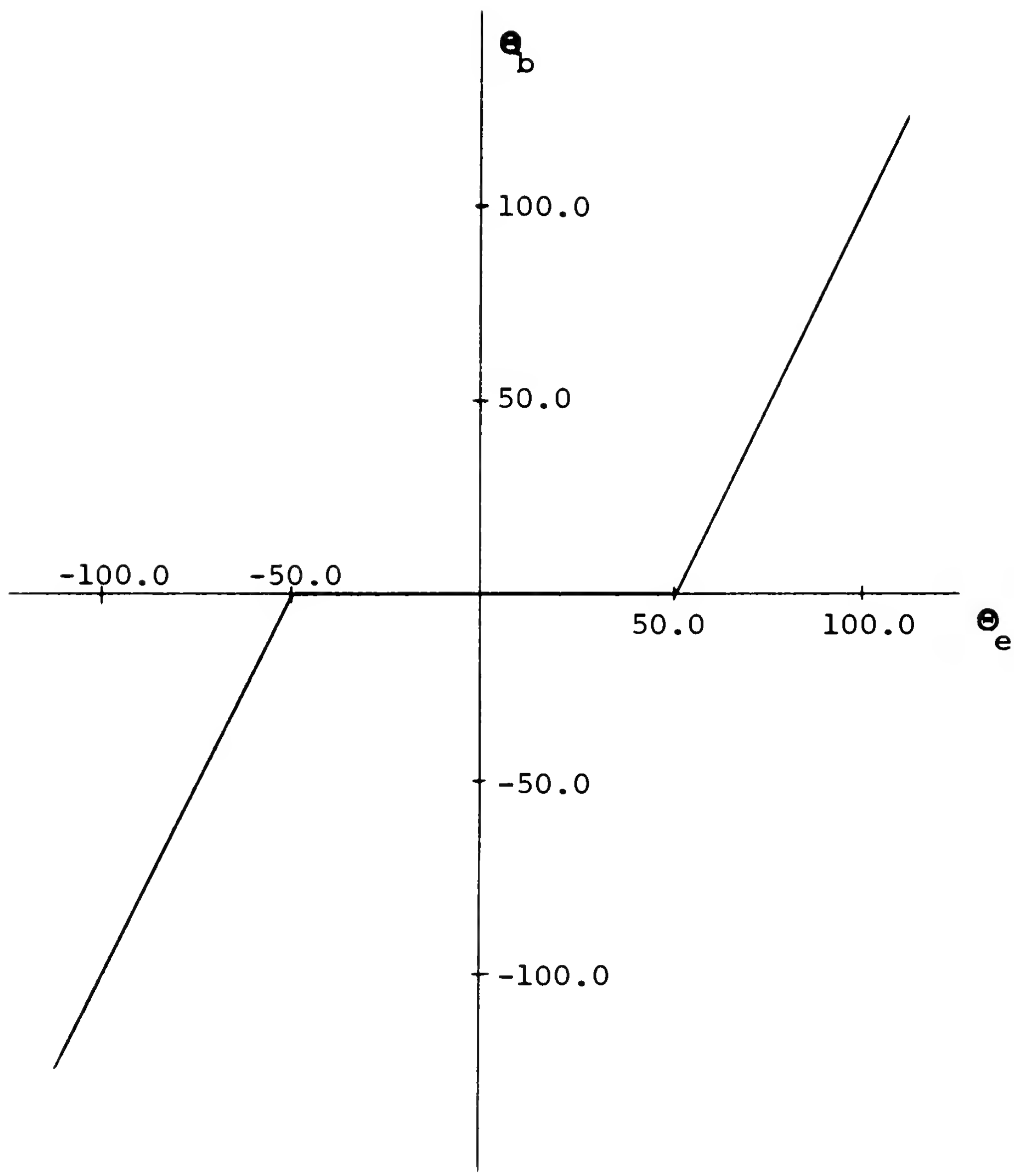

Figure 3.10 Dead zone element of example #3

Dividing the numerator and denominator of the right-hand side of equation (3.16) by Θ_{in} and realizing that when $\Theta_e \geq 50.0$, $\frac{\Theta_e}{\Theta_{in}} = 10.0$, equation (3.16) becomes,

$$\beta = \frac{\frac{2\Theta_e}{\Theta_{in}} - 10.0}{\Theta_e/\Theta_{in}} \qquad \frac{\Theta_e}{\Theta_{in}} \geq 5.0 \qquad (3.17)$$

From equations (3.14) and (3.16) a table of α as a function of $\frac{\Theta_e}{\Theta_{in}}$ is computed. This table is table (III.3) and has $\frac{\Theta_e}{\Theta_{in}}$ in the left-hand column and the corresponding value of α in the right-hand column. The table gives the relationship between α and $\frac{\Theta_e}{\Theta_{in}}$ for a constant $\Theta_{in} = 10.0$.

After finding the relationship between α and $\frac{\Theta_e}{\Theta_{in}}$, the relationship between α and ω is found. Referring to figure (3.9) the transfer function of $\frac{\Theta_e}{\Theta_{in}}$ is,

$$\frac{\Theta_e}{\Theta_{in}} = \frac{14.s^2+145s+304.5}{s^4+13s^3+51s^2+63s+(s^3+10s^2+21s+14.5)\alpha} \qquad (3.18)$$

Using equation (3.18) and the PARAM-5 subprogram, a plot of α versus $\frac{\Theta_e}{\Theta_{in}}$ is made. This plot is figure (3.11) and has ω curves of ω = 0.0, 0.025, 0.05, 0.75, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1.0 plotted on it.

The constant Θ_{in} curve is plotted on figure (3.11) using the coordinates of α and $\frac{\Theta_e}{\Theta_{in}}$ tabulated in table (III.3). The intersection of this constant Θ_{in} curve

$\frac{\Theta_e}{\Theta_{in}}$	α
0.0	0.0
1.0	0.0
2.0	0.0
3.0	0.0
4.0	0.0
5.0	0.0
6.0	0.333
7.0	0.57
8.0	0.75
9.0	0.89
10.0	1.00
11.0	1.09
12.0	1.17
13.0	1.23
14.0	1.28
15.0	1.33
16.0	1.37
17.0	1.41
18.0	1.44
19.0	1.47
20.0	1.50

Table III.3 α as a Function of $\frac{\Theta_e}{\Theta_{in}}$ for the Dead Zone Element of Example #3.

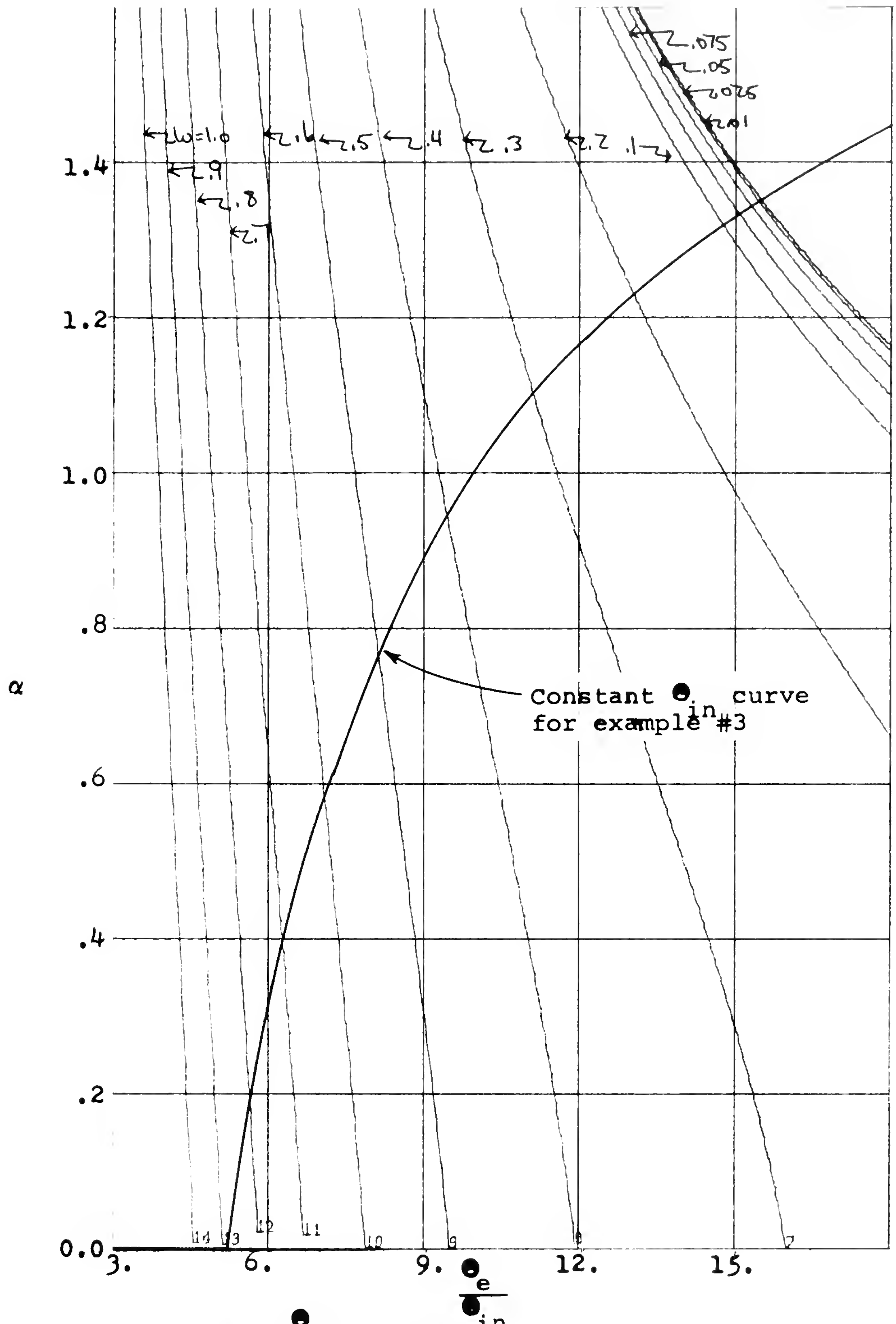

Figure 3.11 α versus $\frac{\Theta_e}{\Theta_{in}}$ magnitude for example #3

and the ω curves, at the associated value of α, gives the relationship between α and ω for $\Theta_{in} = 10.0$.

The final step in the analysis is construction of the closed loop frequency response. Using equation (3.12) in the PARAM-5 subprogram, a plot of $\frac{\Theta_{out}}{\Theta_{in}}$ (dB) versus ω, with constant α lines, is made. This plot is figure (3.12). The constant Θ_{in} curve is replotted onto figure (3.12) from figure (3.11) using α and ω as coordinates. This constant Θ_{in} curve in figure (3.12) is the closed loop frequency response of example #3. An analog simulation of this example was run and the results shown by 'x' on figure (3.12).

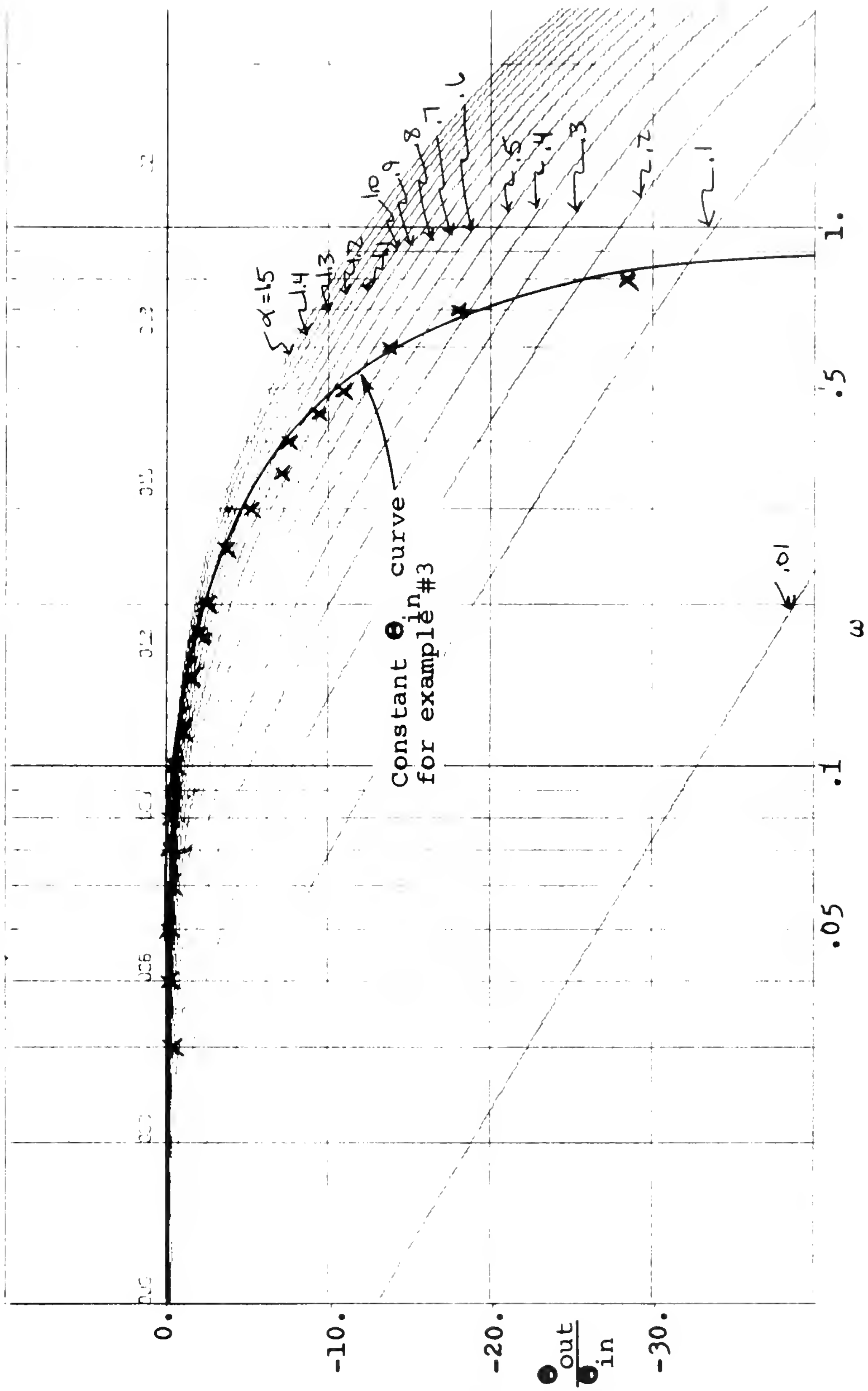

Figure 3.12 Closed loop frequency response of example #3

E. FOURTH EXAMPLE

The fourth example is a fourth order system with a saturation element in a minor feedback path. The analysis is for the closed loop response. Figure (3.13) shows the block diagram of this example where β represents the variable gain of the saturation element in the minor feedback path. Θ_a is the signal going into the saturation element and Θ_d is the signal coming out of the saturation element. Referring to the block diagram of this example(figure(3.13)), the closed loop transfer function is written,

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{14.5}{s^4+13s^3+51s^2+63s+14.5+\beta(s^3+10s^2+21s)} \qquad (3.19)$$

Where β represents the variable gain of the saturation element.

The saturation element for this example is figure (3.14). Θ_a is the signal into the element and Θ_d is the signal out of the element. The saturation element limits at $\Theta_a = 20.0$. For a Θ_a greater than 20.0, the output, Θ_d, is equal to 20.0. From figure (3.13), β may be written as,

$$\beta = \frac{\Theta_d}{\Theta_a} \qquad (3.20)$$

For Θ_a less than 20.0, $\Theta_a = \Theta_d$ and,

$$\beta = \frac{\Theta_a}{\Theta_a} = 1.0 \qquad \Theta_a \leq 20.0 \qquad (3.21)$$

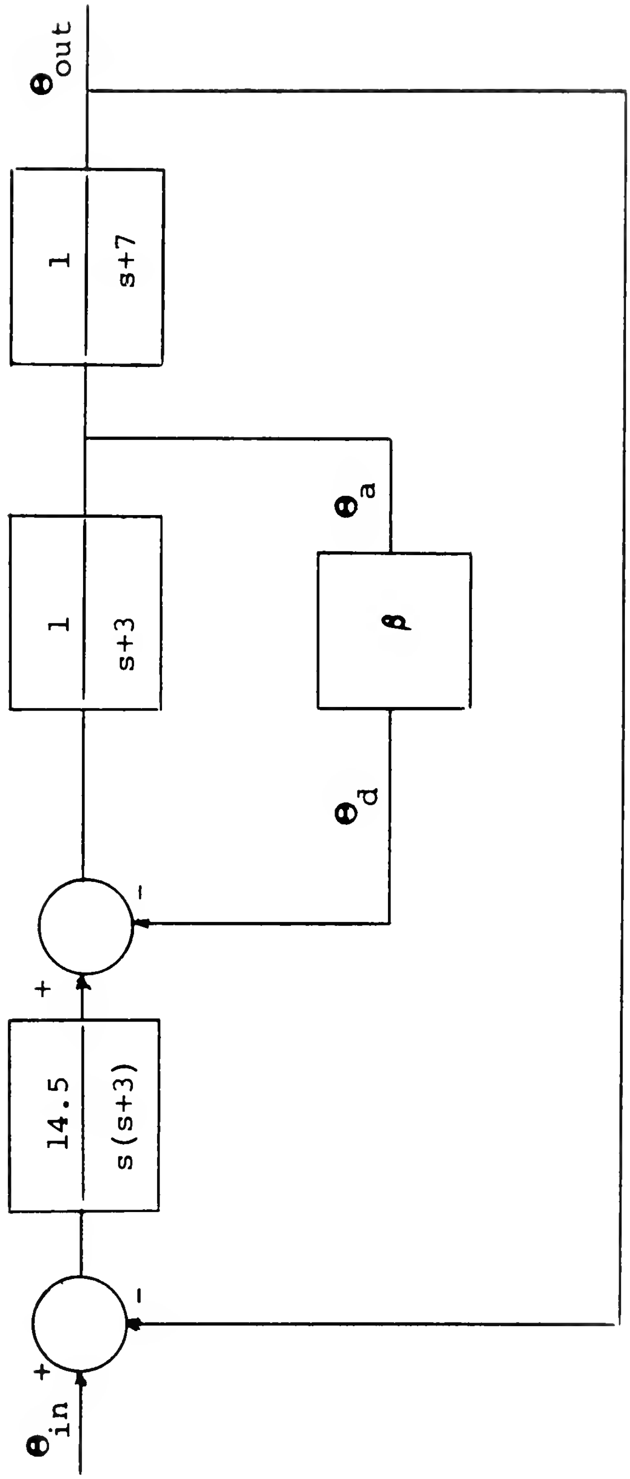

Figure 3.13 Block diagram of example #4 where β represents the variable gain of the saturation element

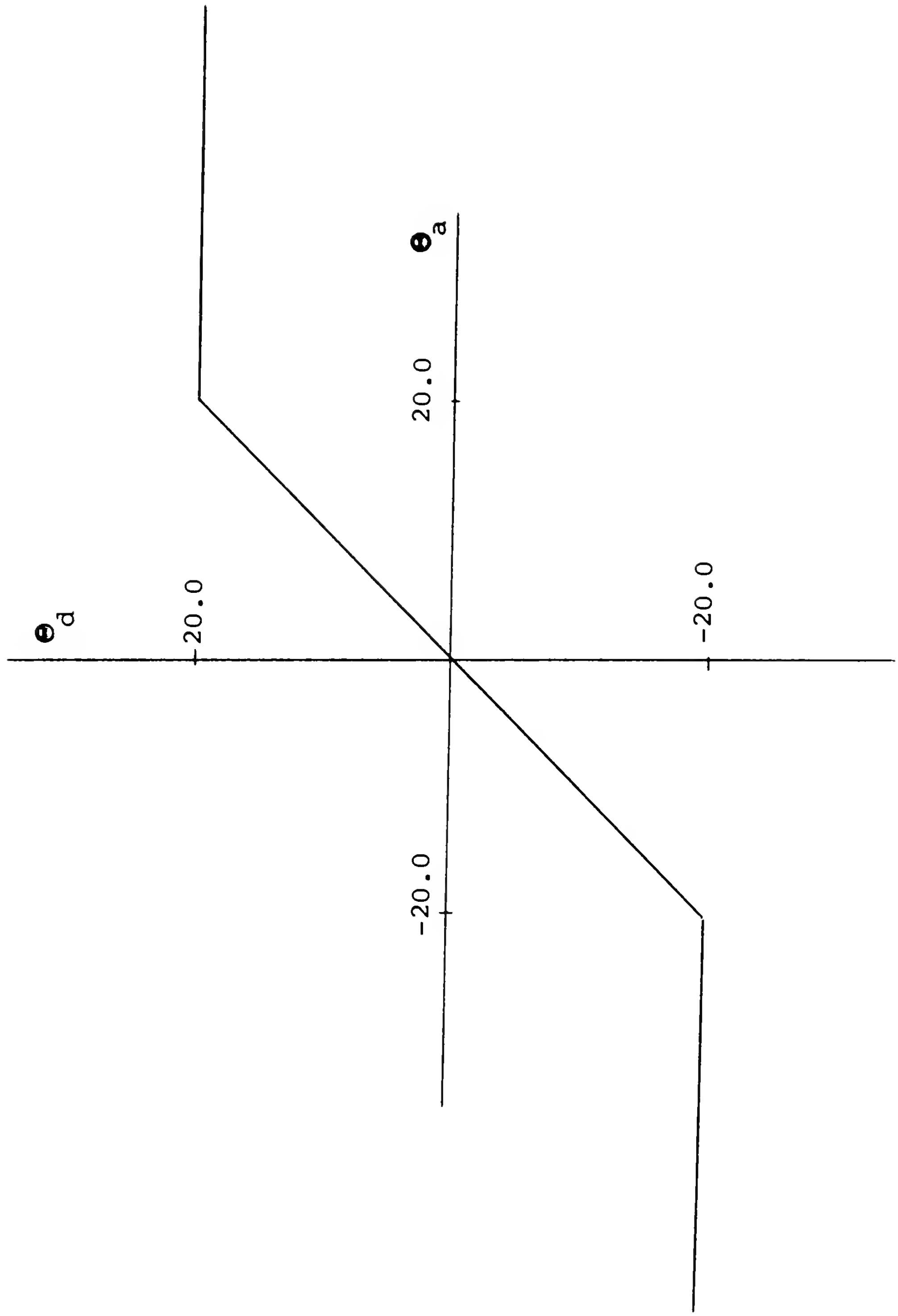

Figure 3.14 Saturation element of example #4

Dividing the limits of equation (3.21) by Θ_{in} when $\Theta_{in} = 10.0$,

$$\beta = 1.0 \qquad \frac{\Theta_a}{\Theta_{in}} \le 2.0 \tag{3.22}$$

For Θ_a greater than 20.0, $\Theta_d = 20.0$,

$$\beta = \frac{\Theta_d}{\Theta_a} = \frac{20.0}{\Theta_a} \qquad \Theta_a \ge 20.0 \tag{3.23}$$

Dividing the numerator and the denominator of the right-hand side of equation (3.23) by Θ_{in} when $\Theta_{in} = 10.0$.

$$\beta = \frac{\frac{20.0}{\Theta_{in}}}{\Theta_a/\Theta_{in}} = \frac{2.0}{\Theta_a/\Theta_{in}} \qquad \frac{\Theta_a}{\Theta_{in}} \ge 2.0 \tag{3.24}$$

Equations (3.22) and (3.24) define the relationship between β and $\frac{\Theta_e}{\Theta_{in}}$ for a constant Θ_{in} equal to 10.0. These equations are put into tabular form and are shown as table (III.4). The left-hand column is $\frac{\Theta_e}{\Theta_{in}}$. Given a certain $\frac{\Theta_e}{\Theta_{in}}$, β is read from the right-hand column.

With the relationship between $\frac{\Theta_e}{\Theta_{in}}$ and β established, the relationship between β and ω is now found. Referring to figure (3.13), $\frac{\Theta_a}{\Theta_{in}}$ is written,

$$\frac{\Theta_a}{\Theta_{in}} = \frac{14.5s+101.5}{s^4+13s^3+51s^2+63s+14.5+\beta(s^3+10s^2+21s)} \tag{3.25}$$

$\frac{\Theta_a}{\Theta_{in}}$	β
0.0	1.0
1.0	1.0
2.0	1.0
3.0	0.667
4.0	0.500
5.0	0.400
6.0	0.333
7.0	0.286
8.0	0.250
9.0	0.222
10.0	0.200

Table III.4 β as a Function of $\frac{\Theta_a}{\Theta_{in}}$ for the Saturation Element of Example #4

Using the equation (3.23) in the PARAM-5 subprogram, a plot of β versus $\frac{\Theta_a}{\Theta_{in}}$ is made. This plot is figure (3.15) and has ω curves of ω = 0.1, 0.2, 0.3, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.9, and 1.0 drawn on it.

With the coordinates of β and $\frac{\Theta_a}{\Theta_{in}}$ from table (III.4), the constant Θ_{in} curve is drawn on figure (3.15). The intersection of this curve with the ω curves at the associated values of β, gives the relationship between β and ω for a constant $\Theta_{in} = 10.0$.

Using equation (3.19) and the PARAM-7 subprogram, a plot of $\frac{\Theta_{out}}{\Theta_{in}}$ versus ω has been made. This plot is figure (3.16) and has β curves of β = 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1.0 drawn on it. Using the coordinates of β and ω, the constant Θ_{in} curve is replotted onto figure (3.16). This constant Θ_{in} curve on figure (3.16) is the closed loop frequency response for example #4. The range of ω shown in figure (3.16) is only over the region in which the response deviates from a constant value of β. The response at frequencies above $\omega = 0.7$ is the same as if the saturation element were replaced by a gain of one. The analog simulation is shown as 'x' on figure (3.16).

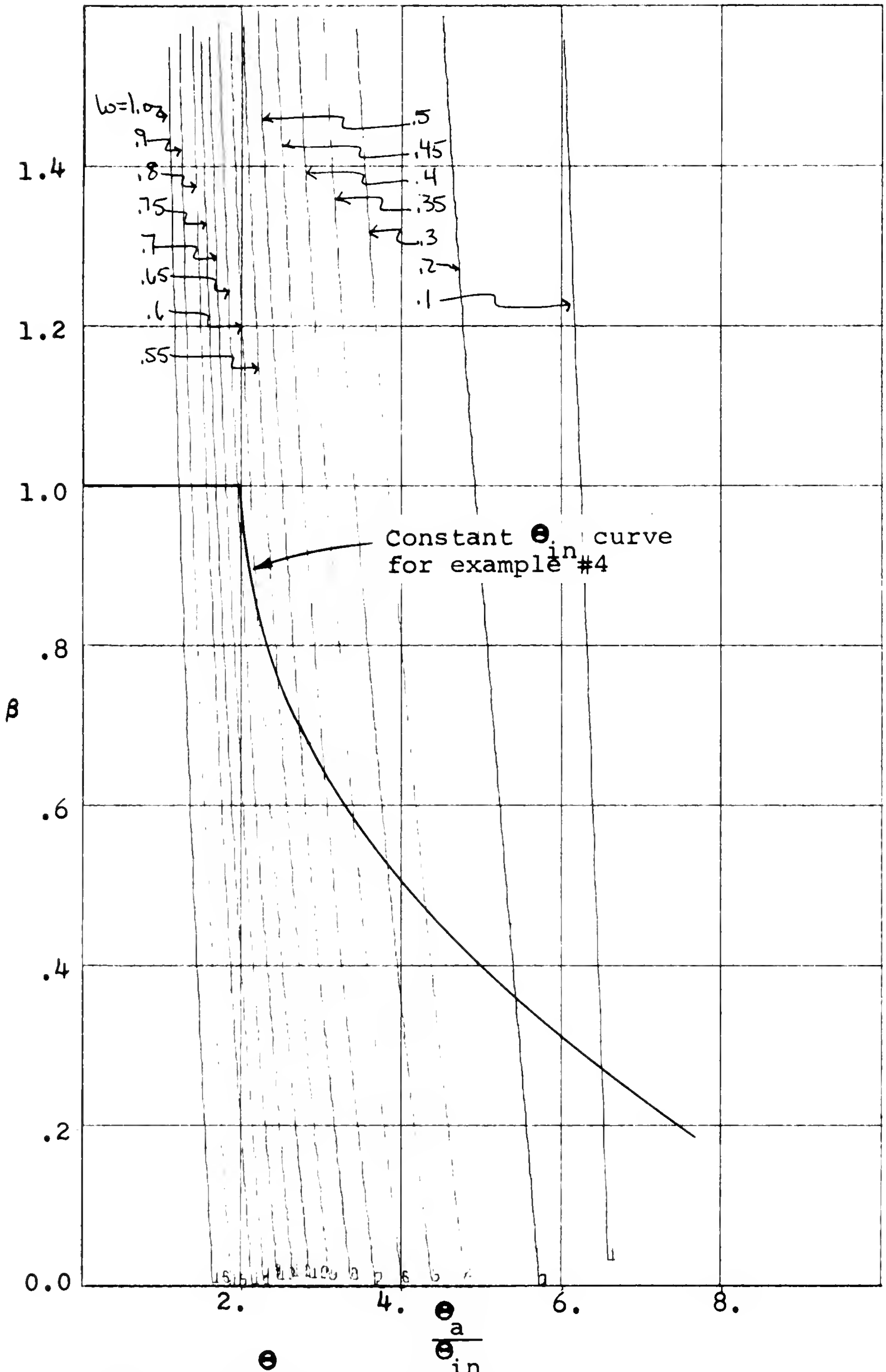

Figure 3.15 β versus $\frac{\theta_a}{\theta_{in}}$ magnitude for example #4

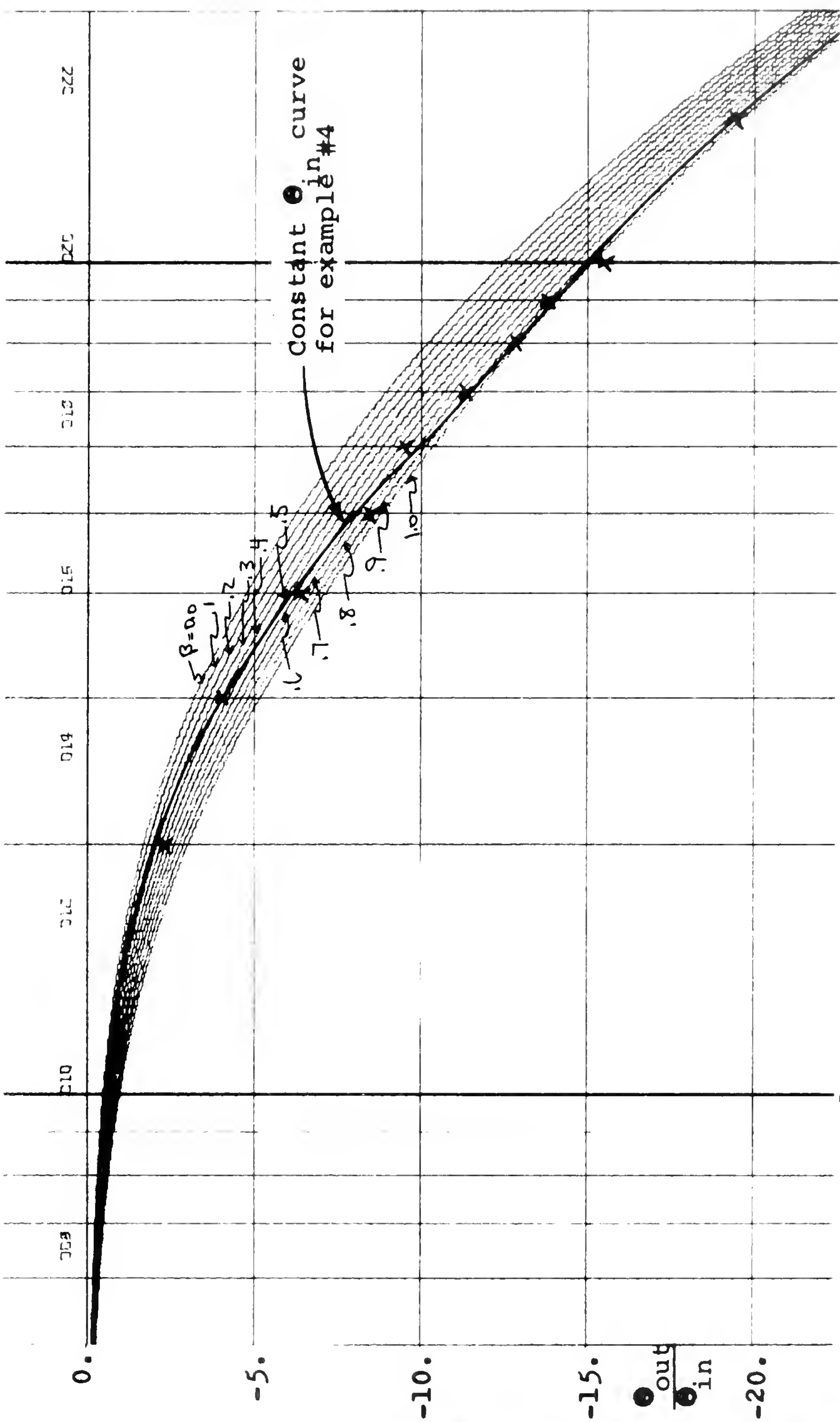

Figure 3.16 Closed loop frequency response for example #4

F. FIFTH EXAMPLE

The fifth example is a type 2 feedback control system for a reproducing contour cutting tool.[3] This system will be analyzed for the open loop frequency response after linear compensation has taken place. The nonlinear element is an ideal relay which has been placed between the compensator and the open loop section of the uncompensated system. The block diagram of this system is figure (3.17) where α represents the variable gain of the relay. The open loop transfer function is,

$$\frac{\Theta_{out}}{\Theta_a} = \frac{225\alpha(s^2+5.6s+7.84)}{s^2(s+5)(s^2+37.5s+350)} \tag{3.26}$$

The nonlinear characteristic of the ideal relay in this example is figure (3.18) where Θ_e is the signal going into the relay and Θ_b is the signal coming out of the relay. For every value of Θ_e, Θ_b has only one value, $\Theta_b = 40$. Referring to figure (3.17), α may be written as,

$$\alpha = \frac{\Theta_b}{\Theta_e} \qquad \Theta_e \geq 0.0 \tag{3.27}$$

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[3] Del Toro, V. and Parker, S.R., Principles of Control Systems Engineering, p. 354, McGraw-Hill, 1960.

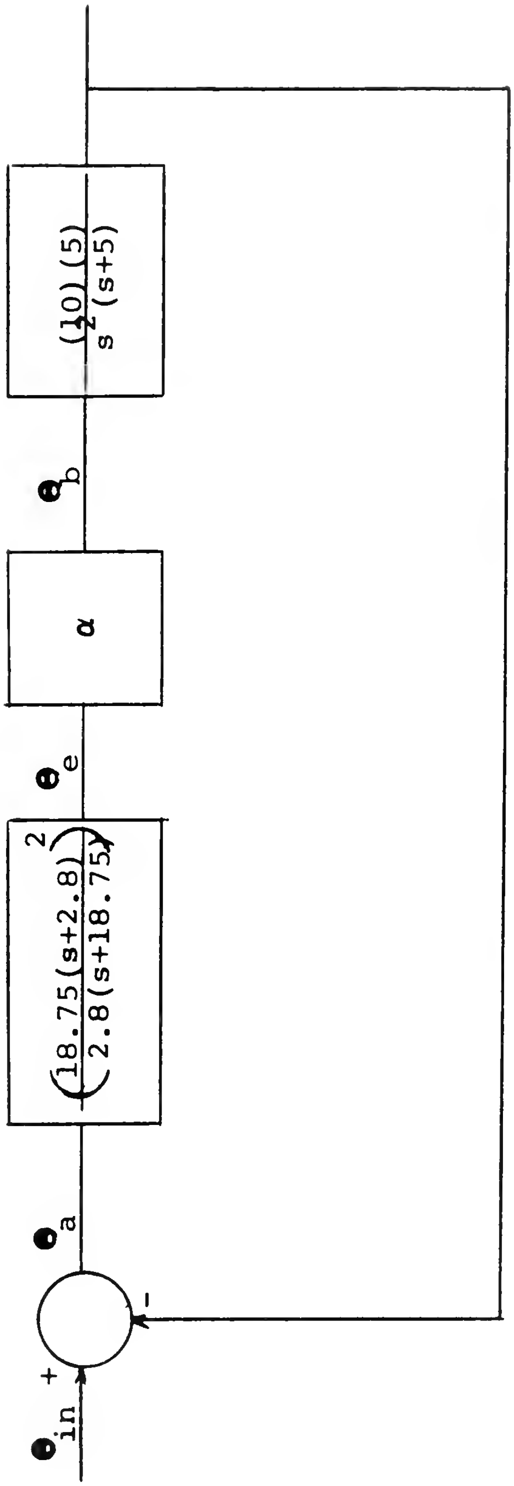

Figure 3.17 Block diagram of example #5 where α represents the variable gain of the relay

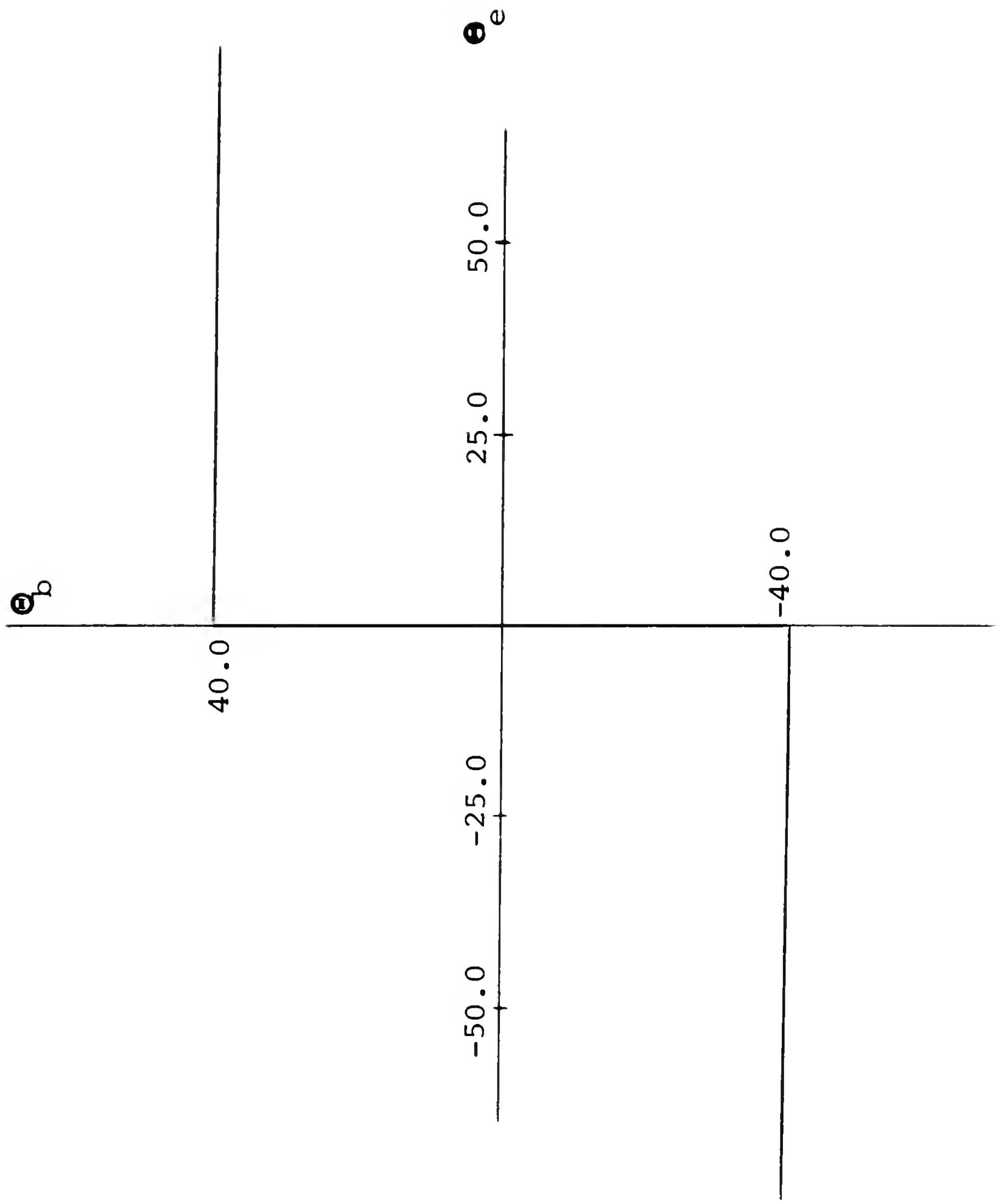

Figure 3.18 Ideal relay of example #5

For $\Theta_b = 40.$, equation (3.27) becomes,

$$\alpha = \frac{40.}{\Theta_e} \qquad \Theta_e \geq 0.0 \qquad (3.28)$$

Dividing the numerator and denominator of the right-hand side of equation (3.28) by Θ_{in} when $\Theta_{in} = 10.0$, equation (3.28) becomes,

$$\alpha = \frac{4.0}{\frac{\Theta_e}{\Theta_{in}}} \qquad \frac{\Theta_e}{\Theta_{in}} \geq 0.0 \qquad (3.29)$$

Equation (3.29) gives the relationship between $\frac{\Theta_e}{\Theta_{in}}$ and α.

From equation (3.29) a table for the relay of α as a function of $\frac{\Theta_e}{\Theta_{in}}$ is made up. This is table (III.5) with $\frac{\Theta_e}{\Theta_{in}}$ as the value in the left-hand column and the associated value of α in the right-hand column. This relationship between α and $\frac{\Theta_e}{\Theta_{in}}$ defines the constant Θ_{in} curve which will be used later.

Because α is a function of $\frac{\Theta_e}{\Theta_{in}}$ for a constant Θ_{in}, a plot of α versus $\frac{\Theta_e}{\Theta_{in}}$ is needed. Referring to figure (3.17), the transfer function $\frac{\Theta_e}{\Theta_{in}}$ is written as,

$$\frac{\Theta_e}{\Theta_{in}} = \frac{4.5s^5+47.7s^4+1610s^3+1768s^2}{s^5+42.5s^4+537.5s^3+1750s^2+\alpha(225s^2+1260s+1765)} \qquad (3.30)$$

$\frac{\Theta_e}{\Theta_{in}}$	α
0.0	∞
0.5	8.0
1.0	4.0
1.5	2.67
2.0	2.00
2.5	1.6
3.0	1.33
3.5	1.14
4.0	1.00
4.5	0.89
5.0	0.80

Table III.5 α as a Function of $\frac{\Theta_e}{\Theta_{in}}$ for the Relay of Example #5

Using equation (3.30) and the PARAM-5 subprogram, a plot of α versus $\frac{\Theta_e}{\Theta_{in}}$ is made. This plot is figure (3.19) and has drawn on it constant ω curves of ω = 0.25, 0.5, 0.75, 1.0, 1.25, 1.75, 2.0, 2.5, 3.0, 4.0, 5.0, 7.5, 20.0 and 40.0.

The constant Θ_{in} curve is drawn on figure (3.19) using the coordinates of α and $\frac{\Theta_e}{\Theta_{in}}$ from table (III.5). The intersection of this constant Θ_{in} curve and the ω curves in figure (3.19) gives the new relationship between α and ω for a Θ_{in} = 10.0. With this final relationship, the open loop frequency response is drawn for the system with the relay.

From equation (3.26) and the PARAM-7 subprogram, a plot of $\frac{\Theta_{out}}{\Theta_a}$ (magnitude - dB and phase) versus ω is made. On this plot α curves are drawn of α = 0.7, 0.85, 1.0, 1.1, 1.25, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 5.0, 6.0, 7.0, 8.0 and 15.0. The $\frac{\Theta_{out}}{\Theta_{in}}$ versus ω plot is figure (3.20). Note that for different values of α, the phase response is the same. This indicates that the nonlinear element does not change the system from its original linear phase response. The α = 1.0 curve is the linear open loop magnitude response. Using the coordinates of α and ω, the constant Θ_{in} curve on figure (3.19) is replotted onto figure (3.20). The constant Θ_{in} curves on figure (3.20) are the open loop magnitude and phase response of example #5 with an ideal relay.

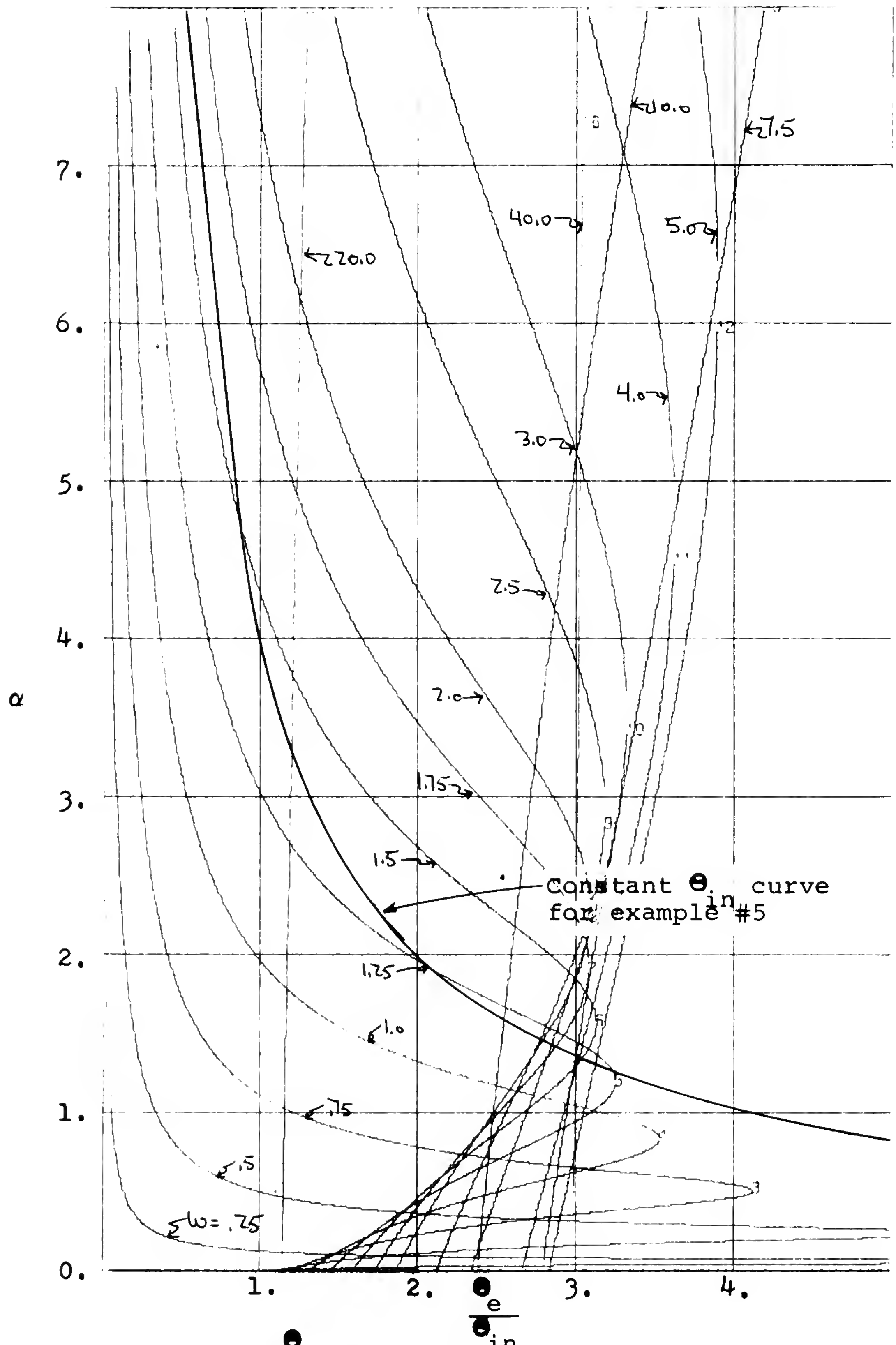

Figure 3.19 α versus $\frac{\Theta_e}{\Theta_{in}}$ for example #5

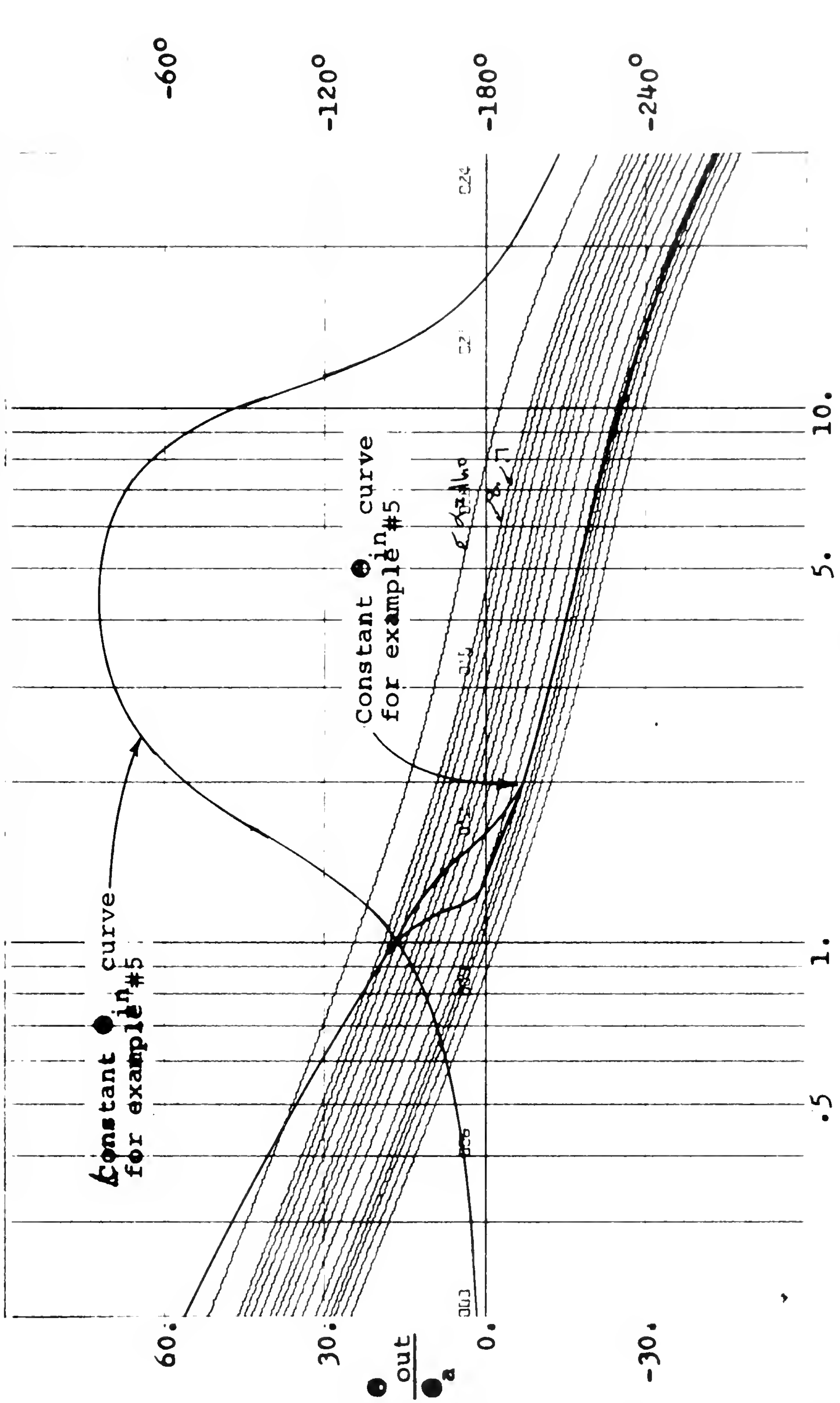

Figure 3.20 Open loop frequency response of example #5

G. SIXTH EXAMPLE

The system for the sixth example is similar to the fifth example. The nonlinear element is coulomb friction plus stiction which converts velocity into torque in a minor feedback loop. The block diagram of the system is figure (3.21) where α represents the coulomb friction plus stiction element. The open loop transfer function is equation (3.31) and is taken from figure (3.21).

$$\frac{\Theta_{out}}{\Theta_a} = \frac{225(s^2+5.6s+7.84)}{s^5+42.5s^4+537.5s^3+1750s^2+\alpha(s^4+42.5s^3+537.5s^2+1750s)} \tag{3.31}$$

The coulomb friction with stiction element is shown in figure (3.22) where $\dot{\Theta}_{out}$ is the signal going into the element and T_1 is the signal coming out of the element. For $\dot{\Theta}_{out}$ less than 2.0, the variable gain can be written as,

$$\alpha = \frac{4.17 - \frac{1}{3}\dot{\Theta}_{out}}{\dot{\Theta}_{out}} \qquad \dot{\Theta}_{out} \le 2.0 \tag{3.32}$$

Dividing the numerator and denominator of the right-hand side of equation (3.32) by Θ_{in} when $\Theta_{in} = 10.0$, equation (3.32) becomes,

$$\alpha = \frac{.417 - \frac{1}{3}\frac{\dot{\Theta}_{out}}{\Theta_{in}}}{\frac{\dot{\Theta}_{out}}{\Theta_{in}}} \qquad \frac{\dot{\Theta}_{out}}{\Theta_{in}} \le 0.2 \tag{3.33}$$

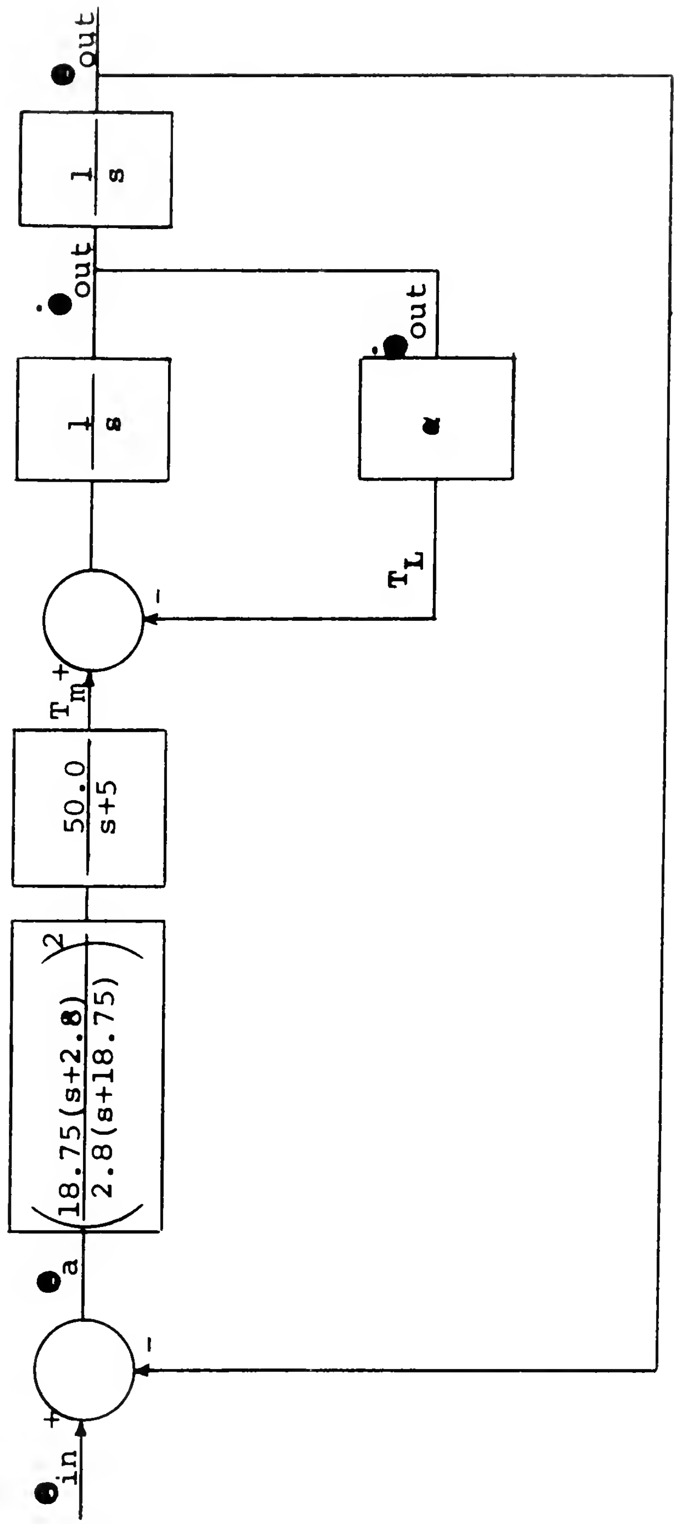

Figure 3.21 Block diagram of example #6 where α represents the variable gain of the coulomb friction plus stiction element

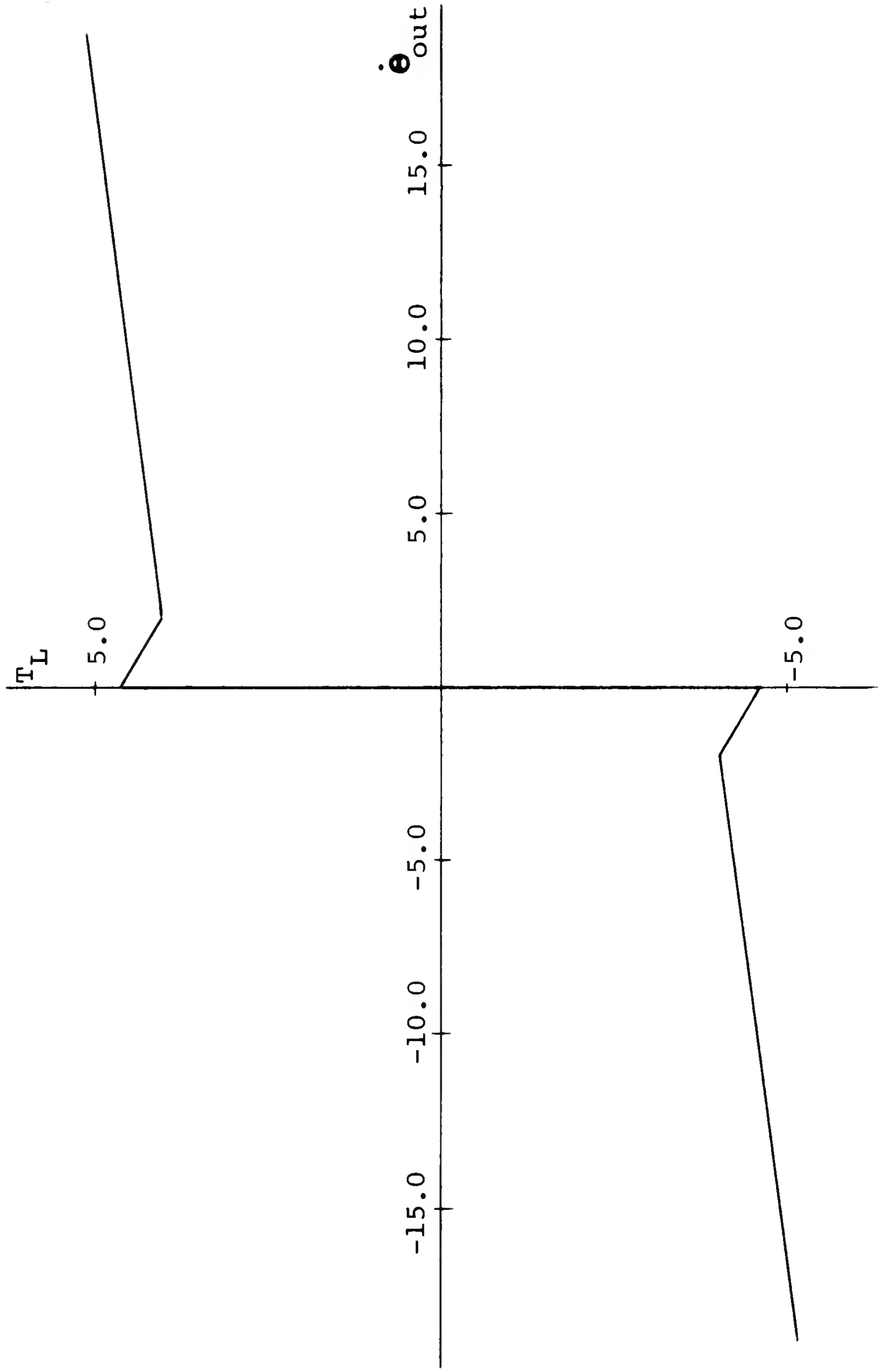

Figure 3.22 Coulomb friction plus stiction element of example #6

For $\dot{\Theta}_{out}$ greater than 2.0, the variable gain can be written as,

$$\alpha = \frac{3.3 + .1\,\dot{\Theta}_{out}}{\dot{\Theta}_{out}} \qquad \dot{\Theta}_{out} \geq 2.0 \qquad (3.34)$$

Dividing the numerator and denominator of the right-hand side of equation (3.34) by $\dot{\Theta}_{in}$ when $\dot{\Theta}_{in} = 10.0$, equation (3.34) becomes,

$$\alpha = \frac{.33 + .1\,\frac{\dot{\Theta}_{out}}{\dot{\Theta}_{in}}}{\dot{\Theta}_{out}/\dot{\Theta}_{in}} \qquad \frac{\dot{\Theta}_{out}}{\dot{\Theta}_{in}} \geq 0.20 \qquad (3.35)$$

Equations (3.33) and (3.34) define the relationship between α and $\frac{\dot{\Theta}_{out}}{\dot{\Theta}_{in}}$ for a constant $\dot{\Theta}_{in} = 10.0$. This relationship is tabulated in table (III.6) where $\frac{\dot{\Theta}_{out}}{\dot{\Theta}_{in}}$ is the left-hand column and α is the right-hand column.

The transfer function of $\frac{\dot{\Theta}_{out}}{\dot{\Theta}_{in}}$ is written from looking at figure (3.21).

$$\frac{\dot{\Theta}_{out}}{\dot{\Theta}_{in}} = \frac{225s^3+1260s^2+1764s}{s^5+42.5s^4+537.5s^3+1975s^2+1260s+1764+\alpha(s^4+42.5s^3+537.5s^2+1750s)} \qquad (3.36)$$

Using equation (3.36) and the PARAM-5 subprogram, a plot of α versus $\frac{\dot{\Theta}_{out}}{\dot{\Theta}_{in}}$ is made. This plot is figure (3.23) and has constant ω curves between $\omega = 0.01$ and $\omega = 10.0$ drawn on it. Using the relationship between α and $\frac{\dot{\Theta}_{out}}{\dot{\Theta}_{in}}$ from

$\dfrac{\dot{\Theta}_{out}}{\dot{\Theta}_{in}}$	α
.00	∞
.05	8.0
.10	3.84
.15	2.45
.20	1.75
.3	1.2
.4	.95
.5	.76
.6	.65
.7	.57
.8	.51
.9	.466
1.0	.43
1.1	.40
1.2	.375
1.3	.354
1.4	.335
1.5	.32

Table III.6 α as a Function of $\dfrac{\dot{\Theta}_{out}}{\dot{\Theta}_{in}}$ for Coulomb Friction Plus Stiction in Example #6

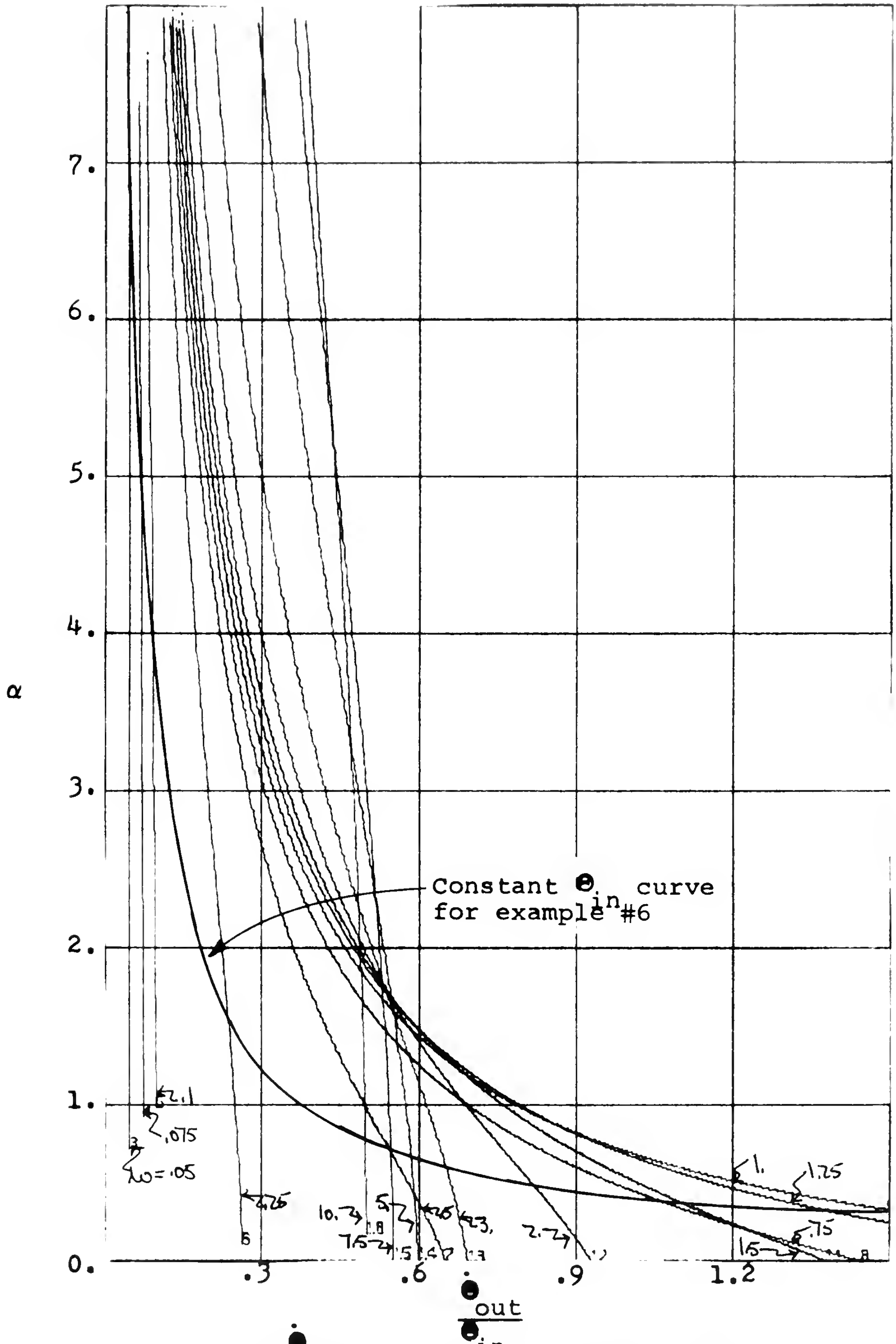

Figure 3.23 α versus $\frac{\dot{\Theta}_{out}}{\dot{\Theta}_{in}}$ magnitude for example #6

table (III.6), the constant Θ_{in} curve is drawn on figure (3.23). The intersection of the constant Θ_{in} curve with the ω curves in figure (3.23) gives the relationship between α and ω for the coulomb friction plus stiction element.

The final step in the open loop frequency response analysis is making a plot of $\frac{\Theta_{out}}{\Theta_a}$ versus ω. Using equation (3.31) and the PARAM-7 subprogram, plots of $\frac{\Theta_{out}}{\Theta_a}$ magnitude versus ω (figure (3.24)) and $\frac{\Theta_{out}}{\Theta_a}$ - phase versus ω (figure (3.25)) are made. Both plots have constant α curves drawn on them. Using the coordinates of α and ω, the constant Θ_{in} curve in figure (3.23) is replotted onto figures (3.24) and (3.25). The constant Θ_{in} curves on these two plots are the open loop magnitude and phase frequency response of this example. The linear response (α = 0.0) is also shown.

For a better analysis, the closed loop response is shown in figure (3.26) for the magnitude and figure (3.27) for the phase. These plots were obtained in the same manner as previous examples. The linear response (α = 0.0) is also shown.

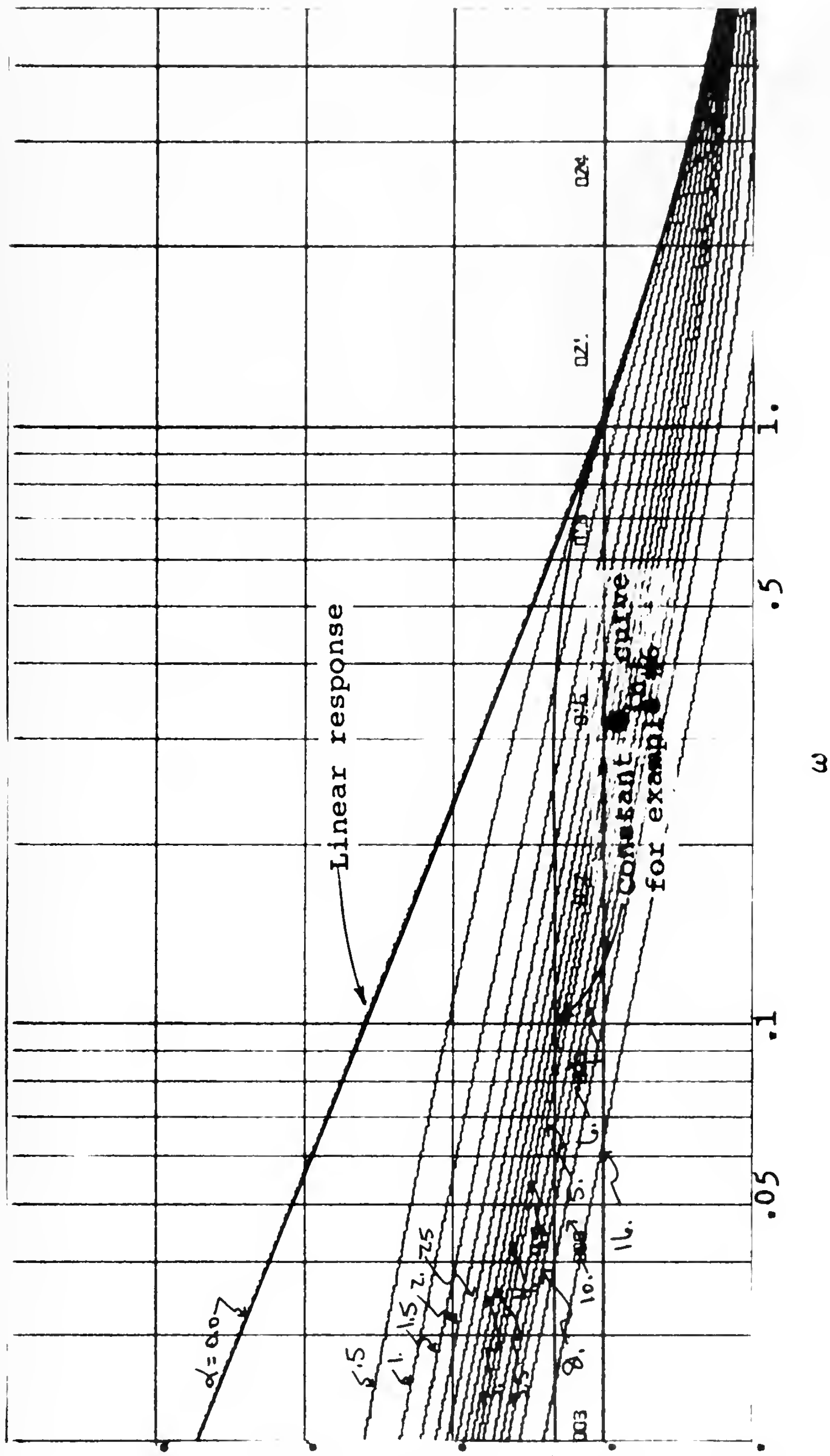

Figure 3.24 Open loop magnitude frequency response of example #6

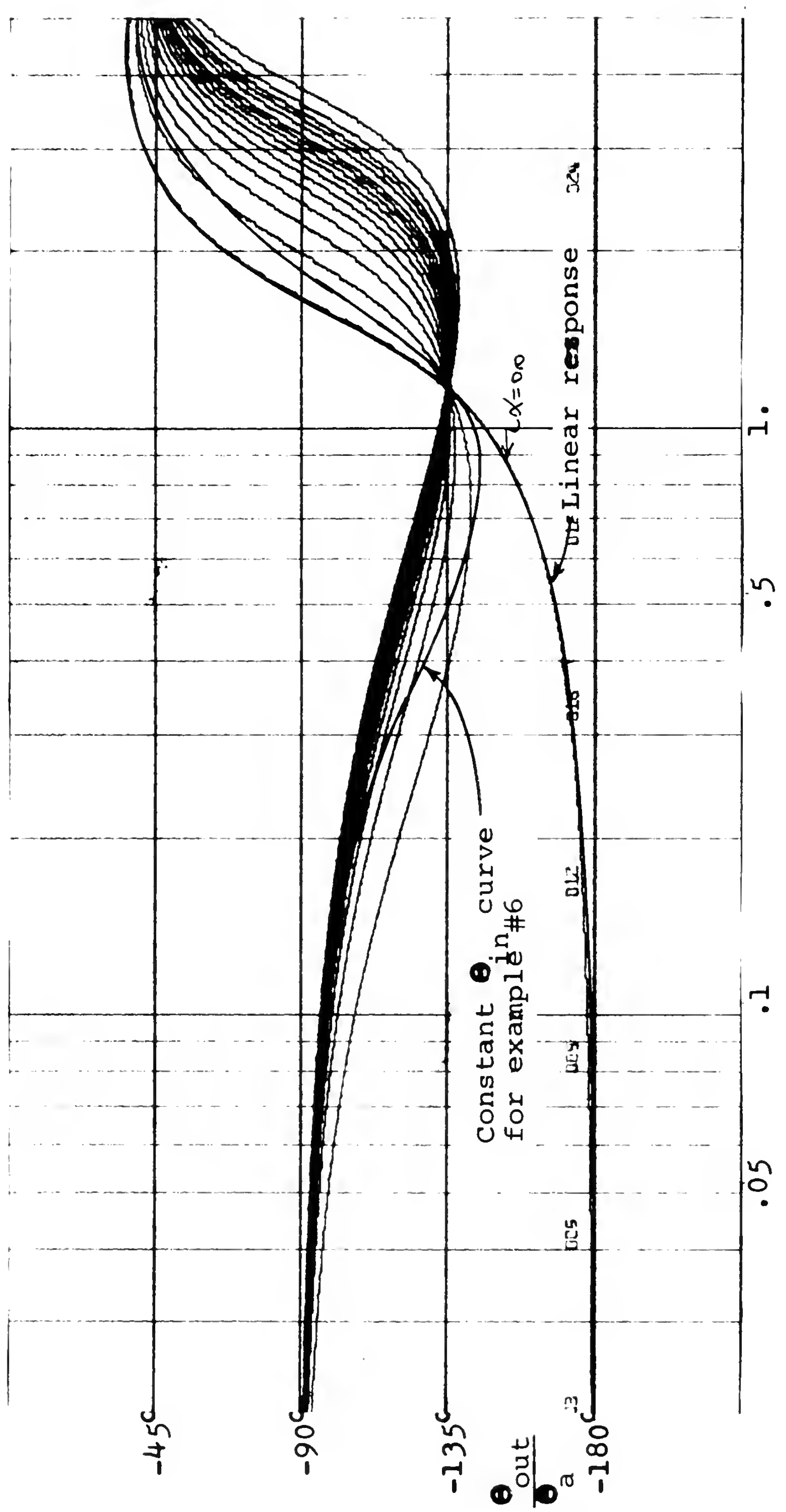

Figure 3.25 Open loop phase frequency response of example #6

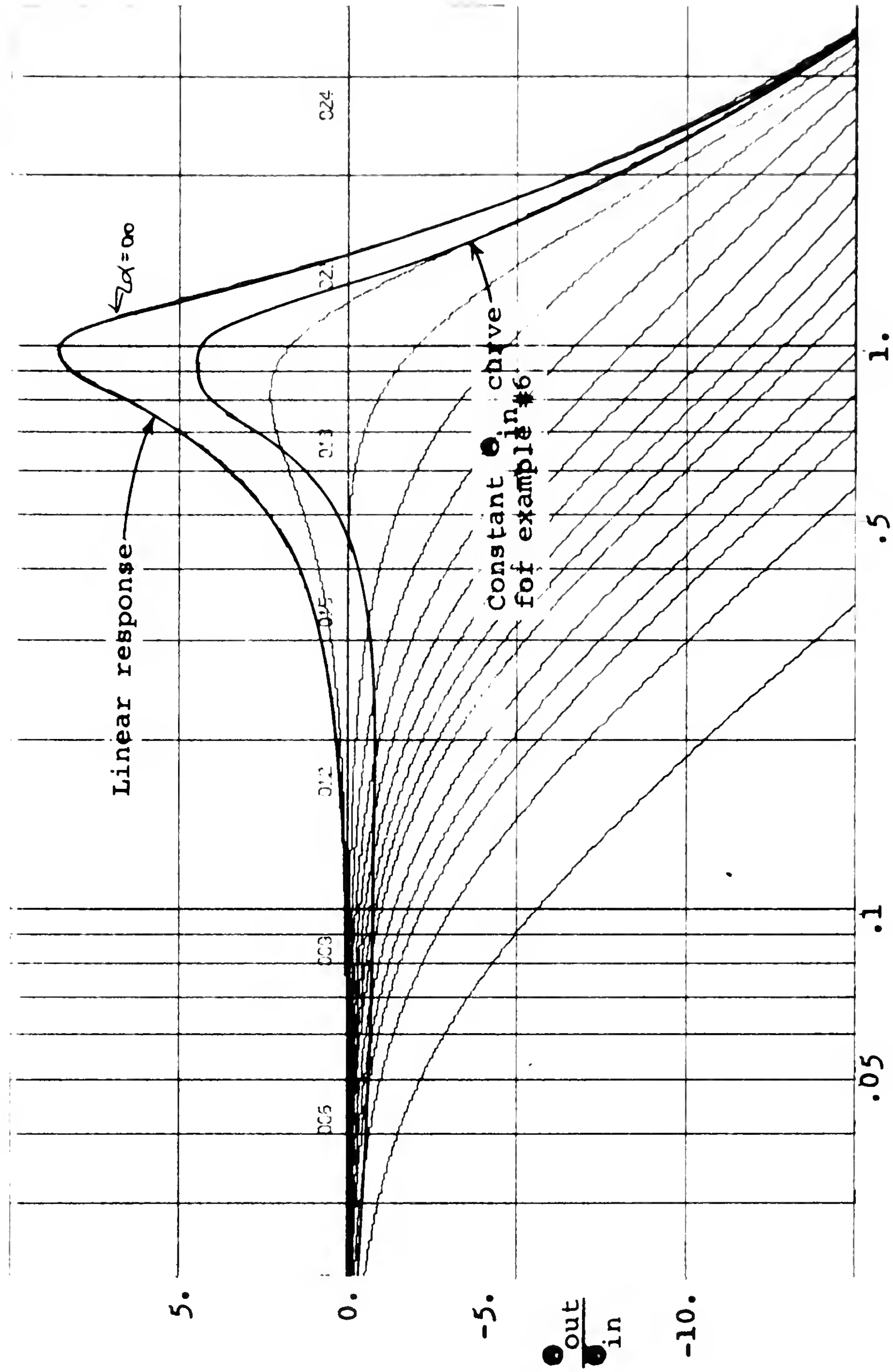

Figure 3.26 Closed loop magnitude frequency response of example #6

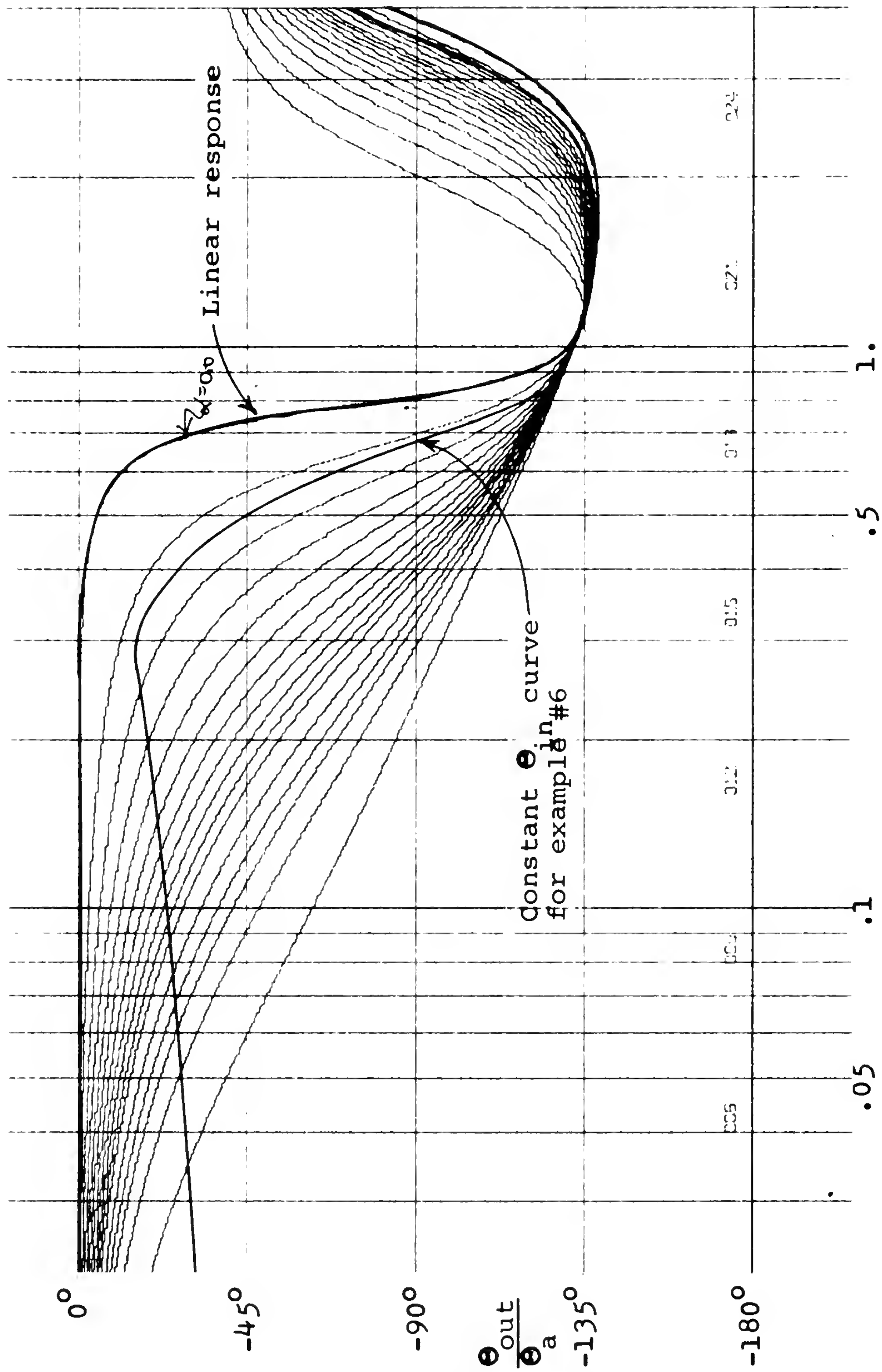

Figure 3.27 Closed loop phase frequency response of example #6

IV. THE EFFECT OF CHANGING THE INPUT

A. INTRODUCTION

In the past examples, the frequency response was described as a constant Θ_{in} curve. The frequency response was obtained by letting Θ_{in} be equal to some specified value. A change in Θ_{in} results in a change in the response. In this chapter, two examples are analyzed for different values of input. The first example has a saturation element in the forward path, and the second example has a dead zone element in the major feedback path.

B. THE EFFECT OF CHANGING THE INPUT OF A SYSTEM WITH A SATURATION ELEMENT

The example of chapter 2 used an input of magnitude 10.0, $\Theta_{in} = 10.0$. To see what is the effect of various other amplitudes of signals, Θ_{in} of 5.0, 7.5, 10.0, 15.0, 20.0 and 25.0 will be used. The steps in this analysis are the same as those taken in previous examples. In order to derive a relationship between gain of the saturation element and frequency, the transfer function of the signal going into the saturation element must be obtained. By simple manipulation $\frac{\Theta_e}{\Theta_{in}}$ can be found, as shown in the following steps:

$$\Theta_e = \Theta_{in} - \Theta_{out} \tag{4.1}$$

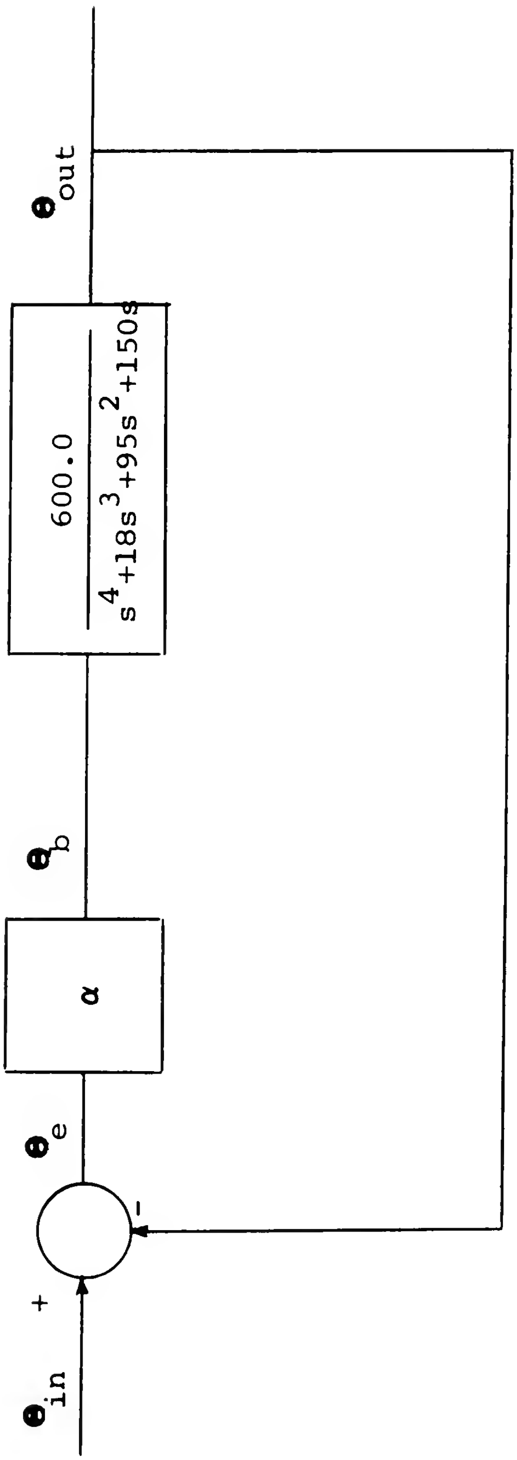

Figure 4.1 Block diagram of servomechanism in chapter 2

Dividing by Θ_{in},

$$\frac{\Theta_e}{\Theta_{in}} = 1 - \frac{\Theta_{out}}{\Theta_{in}}$$

$$= 1 - \frac{600\alpha}{s^4+18s^3+95s^2+150s+600\alpha} \tag{4.2b}$$

$$= \frac{s^4+18s^3+95s^2+150s}{s^4+18s^3+95s^2+150s+600\alpha} \tag{4.2c}$$

where α is the variable gain of the saturation element.

By using the PARAM program mentioned previously, a plot of α, (variable gain of the saturation element), versus the magnitude of $\frac{\Theta_e}{\Theta_{in}}$ is obtained. This is shown in figure (4.2). The saturation element may be represented by a variable gain which is a function of the input signal, Θ_e. However the plot in figure (4.2) is gain, α, versus $\frac{\Theta_e}{\Theta_{in}}$. By the following equations, a relationship between α and $\frac{\Theta_e}{\Theta_{in}}$ is developed.

$$\alpha = \frac{\Theta_b}{\Theta_e} \tag{4.3}$$

where Θ_b is the signal out of the saturation element as shown in figure (4.3).

Dividing both the numerator and the denominator of the right-hand side of equation (4.3) by Θ_{in},

$$\alpha = \frac{\Theta_b/\Theta_{in}}{\Theta_e/\Theta_{in}} \tag{4.4}$$

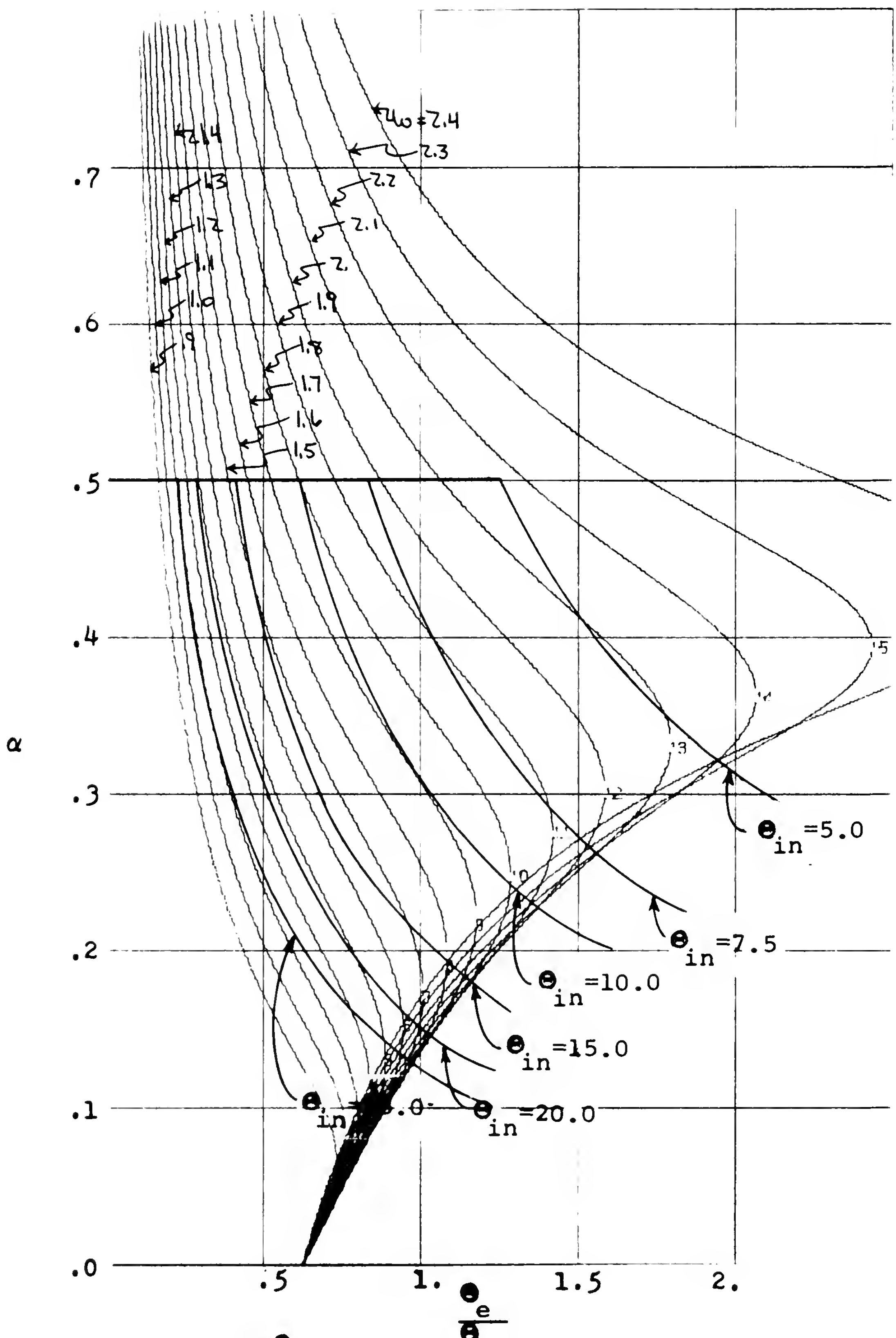

Figure 4.2 α versus $\frac{\Theta_e}{\Theta_{in}}$ with constant ω curves and constant Θ_{in} curves

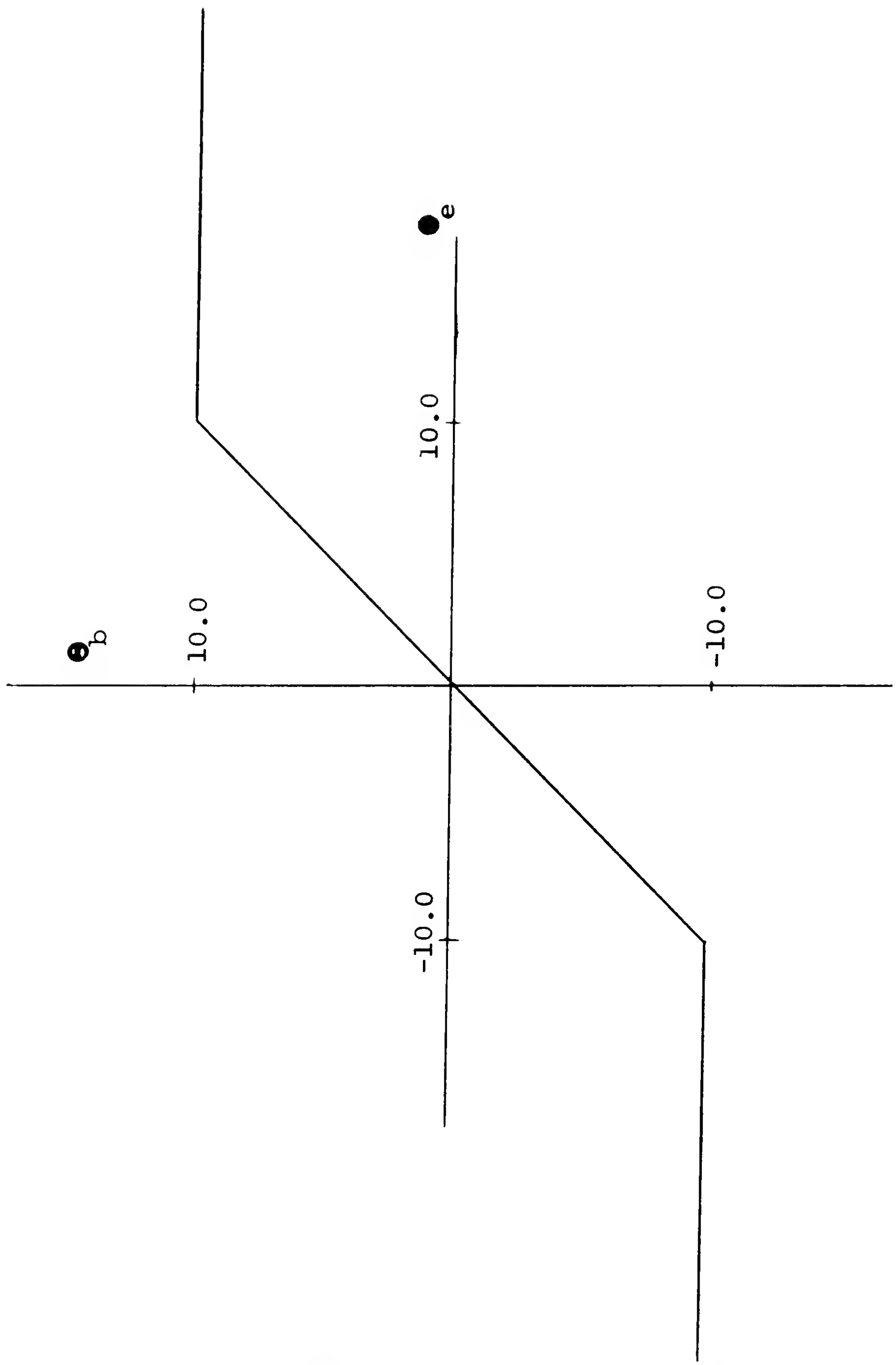

Figure 4.3 Saturation element in servomechanism

α is constant in the region where $\Theta_e \leq 10.0$. In the region where $\Theta_e \geq 10.0$, Θ_b, the output of the saturation element, is 10.0. Thus equation (4.4) becomes,

$$\alpha = \frac{10.0/\Theta_{in}}{\Theta_e/\Theta_{in}} \qquad \Theta_e \geq 10.0 \qquad (4.5)$$

For $\Theta_{in} = 5.0$, equation (4.4) becomes

$$\alpha = 1.0 \qquad \frac{\Theta_e}{\Theta_{in}} \leq 2.0 \qquad (4.62)$$

$$\alpha = \frac{2.0}{\Theta_e/\Theta_{in}} \qquad \frac{\Theta_e}{\Theta_{in}} \geq 2.0 \qquad (4.6b)$$

In like manner, for $\Theta_{in} = 7.5$

$$\alpha = 1.0 \qquad \frac{\Theta_e}{\Theta_{in}} \leq 1.333 \qquad (4.7a)$$

$$\alpha = \frac{1.333}{\Theta_e/\Theta_{in}} \qquad \frac{\Theta_e}{\Theta_{in}} \geq 1.333 \qquad (4.7b)$$

For $\Theta_{in} = 10$

$$\alpha = 1.0 \qquad \frac{\Theta_e}{\Theta_{in}} \leq 1.0 \qquad (4.8a)$$

$$\alpha = \frac{1.0}{\Theta_e/\Theta_{in}} \qquad \frac{\Theta_e}{\Theta_{in}} \geq 1.0 \qquad (4.8b)$$

For $\Theta_{in} = 20.0$

$$\alpha = 1.0 \qquad \frac{\Theta_e}{\Theta_{in}} \leq 0.50 \tag{4.9a}$$

$$\alpha = \frac{0.5}{\Theta_e/\Theta_{in}} \qquad \frac{\Theta_e}{\Theta_{in}} \geq 0.50 \tag{4.9b}$$

For $\Theta_{in} = 15.0$

$$\alpha \;\; 1.0 \qquad \frac{\Theta_e}{\Theta_{in}} \leq 0.667 \tag{4.10a}$$

$$\alpha = \frac{0.667}{\Theta_e/\Theta_{in}} \qquad \frac{\Theta_e}{\Theta_{in}} \geq 0.667 \tag{4.10b}$$

For $\Theta_{in} = 25.0$

$$\alpha = 1.0 \qquad \frac{\Theta_e}{\Theta_{in}} \leq 0.40 \tag{4.11a}$$

$$\alpha = \frac{0.40}{\Theta_e/\Theta_{in}} \qquad \frac{\Theta_e}{\Theta_{in}} \geq 0.40 \tag{4.11b}$$

By using equations (4.6), (4.7), (4.8), (4.9), (4.10) and (4.11), a table may be made up of β as a function of $\frac{\Theta_e}{\Theta_{in}}$ and Θ_{in}. This is shown in table (IV.1).

Using the values of α in table (IV.1), the constant Θ_{in} curves are drawn on figure (4.2), using α and $\frac{\Theta_e}{\Theta_{in}}$ as coordinates.

$\frac{\Theta_e}{\Theta_{in}}$	α $\Theta_{in}=5.0$	$\Theta_{in}=7.5$	$\Theta_{in}=10.0$	$\Theta_{in}=15.0$	$\Theta_{in}=20.0$	$\Theta_{in}=25.0$
0.0	1.0	1.0	1.0	1.0	1.0	1.0
0.1	1.0	1.0	1.0	1.0	1.0	1.0
0.2	1.0	1.0	1.0	1.0	1.0	1.0
0.3	1.0	1.0	1.0	1.0	1.0	1.0
0.4	1.0	1.0	1.0	1.0	1.0	1.0
0.5	1.0	1.0	1.0	1.0	1.0	0.8
0.6	1.0	1.0	1.0	1.0	0.835	0.667
0.7	1.0	1.0	1.0	0.955	0.715	0.572
0.8	1.0	1.0	1.0	0.835	0.625	0.50
0.9	1.0	1.0	1.0	0.742	0.556	0.445
1.0	1.0	1.0	1.0	0.667	0.5	0.4
1.0	1.0	1.0	0.91	0.606	0.455	0.364
1.2	1.0	1.0	0.833	0.556	0.417	0.334
1.3	1.0	1.0	0.77	0.514	0.385	0.308
1.4	1.0	0.955	0.714	0.477	0.357	0.286
1.5	1.0	0.89	0.667	0.445	0.333	0.267
1.6	1.0	0.835	0.625	0.417	0.312	0.25
1.7	1.0	0.785	0.588	0.393	0.294	0.235
1.8	1.0	0.741	0.555	0.371	0.278	0.222
1.9	1.0	0.702	0.526	0.352	0.263	0.212
2.0	1.0	0.669	0.500	0.334	0.250	0.200
2.1	0.95	0.633	0.476			
2.2	0.91	0.606	0.455			
2.3	0.87	0.55	0.435			
2.4	0.83	0.536	0.433			
2.5	0.80	0.534	0.417			
2.6	0.77	0.514	0.400			
2.7	0.74	0.494				
2.8	0.715	0.476				
2.9	0.69	0.460				
3.0	0.667	0.445				

Table IV.1 α as a Function of $\frac{\Theta_e}{\Theta_{in}}$ and Θ_{in} for the Saturation Element

The intersection of the constant Θ_{in} curves and ω curves gives the relationship between α, the variable gain of the saturation element, and ω, frequency. It should be noted that each constant Θ_{in} curve intersects a particular constant ω curve two or three times, giving two or three values of α for the same frequency. This is the jump resonance effect which will be shown more clearly in the closed loop frequency response curves.

Using the PARAMS program again, a plot of $\frac{\Theta_{out}}{\Theta_{in}}$ versus ω is obtained. This plot, figure (4.4), consists of constant α curves of various values between $\alpha = 0.25$ and $\alpha = 1.0$. The coordinates of the constant Θ_{in} curves in figure (4.2) are ω, frequency, and α, nonlinear gain. Using these coordinates, the constant Θ_{in} curves are replotted in figure (4.4). As was noted previously in figure (4.2), each constant Θ_{in} curve intersects some ω curves more than once. When plotted on figure (4.4), this effect is clearly seen as being a jump resonance effect caused by the saturation element.

The effect of changing the input is seen in figure (4.4). Increasing Θ_{in} decreases the magnitude of the resonance peak and decreases the resonance frequency. The jump resonance remains about the same as Θ_{in} changes.

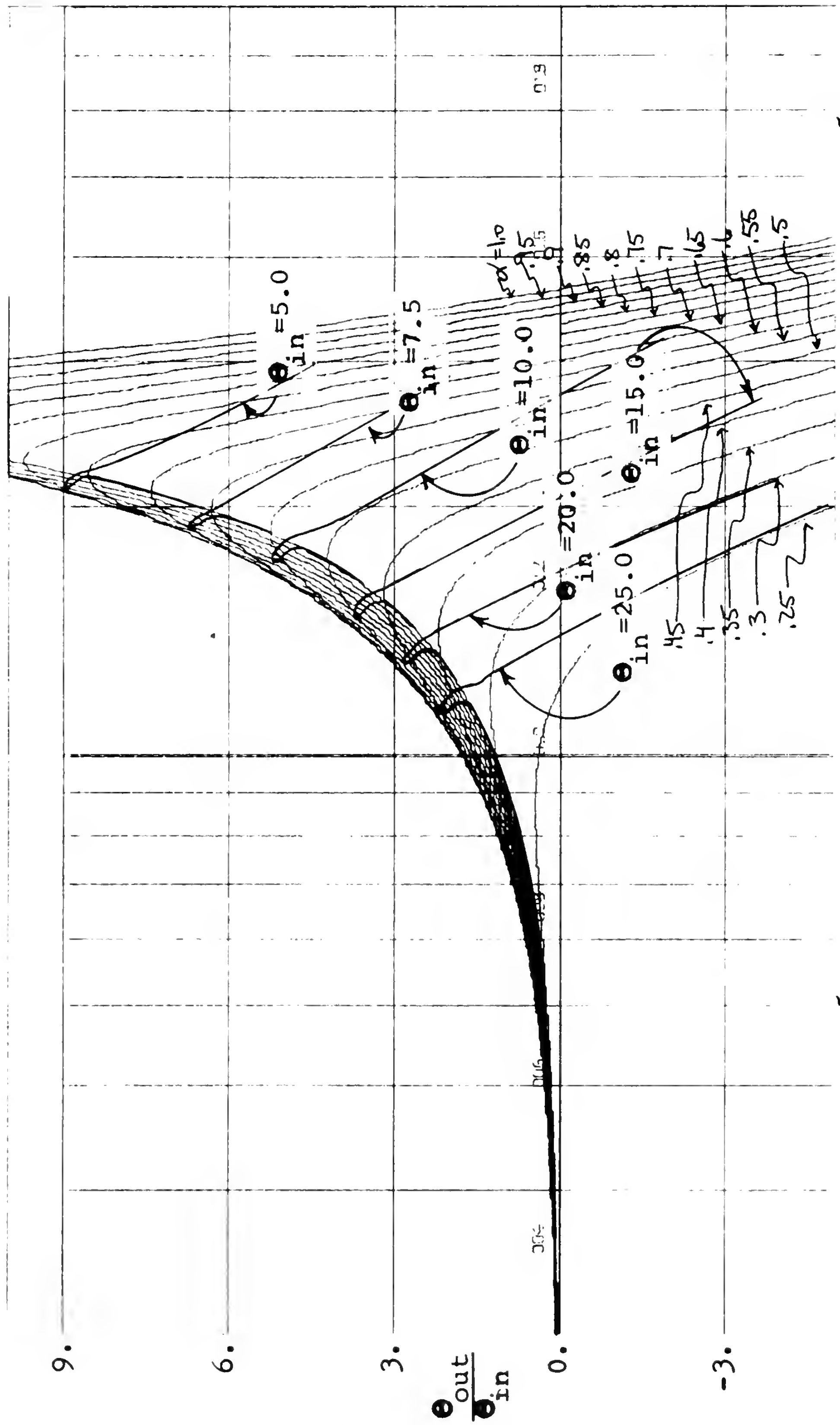

Figure 4.4 Closed loop frequency response for various Θ_{in}

C. THE EFFECT OF CHANGING THE INPUT OF A SYSTEM WITH A DEAD ZONE ELEMENT

The effect of changing the input of the system with a dead zone element can be seen in the same manner as the previous example. The block diagram of the system with dead zone is figure (4.5), where β is the variable gain of the dead zone element. The dead zone element is different from the one used in chapter 2. Its input-output describing curve is shown in figure (4.6) where Θ_{out} is the signal out of the element. Note that the non-dead zone portion of the curve does not have unity gain. The gain is 2.0 when the signal is not in the dead zone portion.

The frequency of the system will be analyzed for four different inputs. These inputs are $\Theta_{in} = 10.0$. $\Theta_{in} = 20.0$,

$\Theta_{in} = 40.0$ and $\Theta_{in} = 80.0$. In order to analyze the problem, the same steps must be taken as in the preceding examples. The first step is to find a relationship between $\frac{\Theta_{out}}{\Theta_{in}}$ and β, the nonlinear variable gain of the saturation element.

Referring to figure (4.5), it is seen that,

$$\beta = \frac{\Theta_a}{\Theta_{out}}$$

However $\beta = 0.0$ when $\Theta_{out} \leq 40.0$ from figure (4.6). Or,

$$\Theta_a = 2(\Theta_{out}-40) = 2\Theta_{out}-80. \qquad \Theta_{out} \geq 40.0 \qquad (4.13a)$$

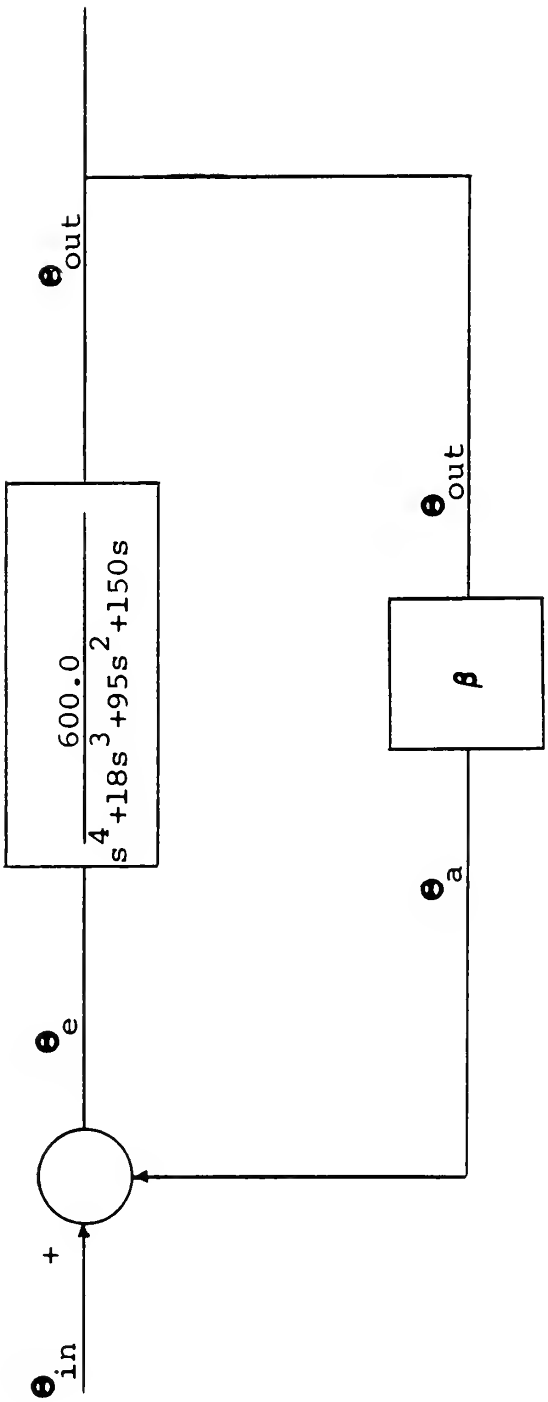

Figure 4.5 Block diagram of servomechanism with dead zone element in the feedback path

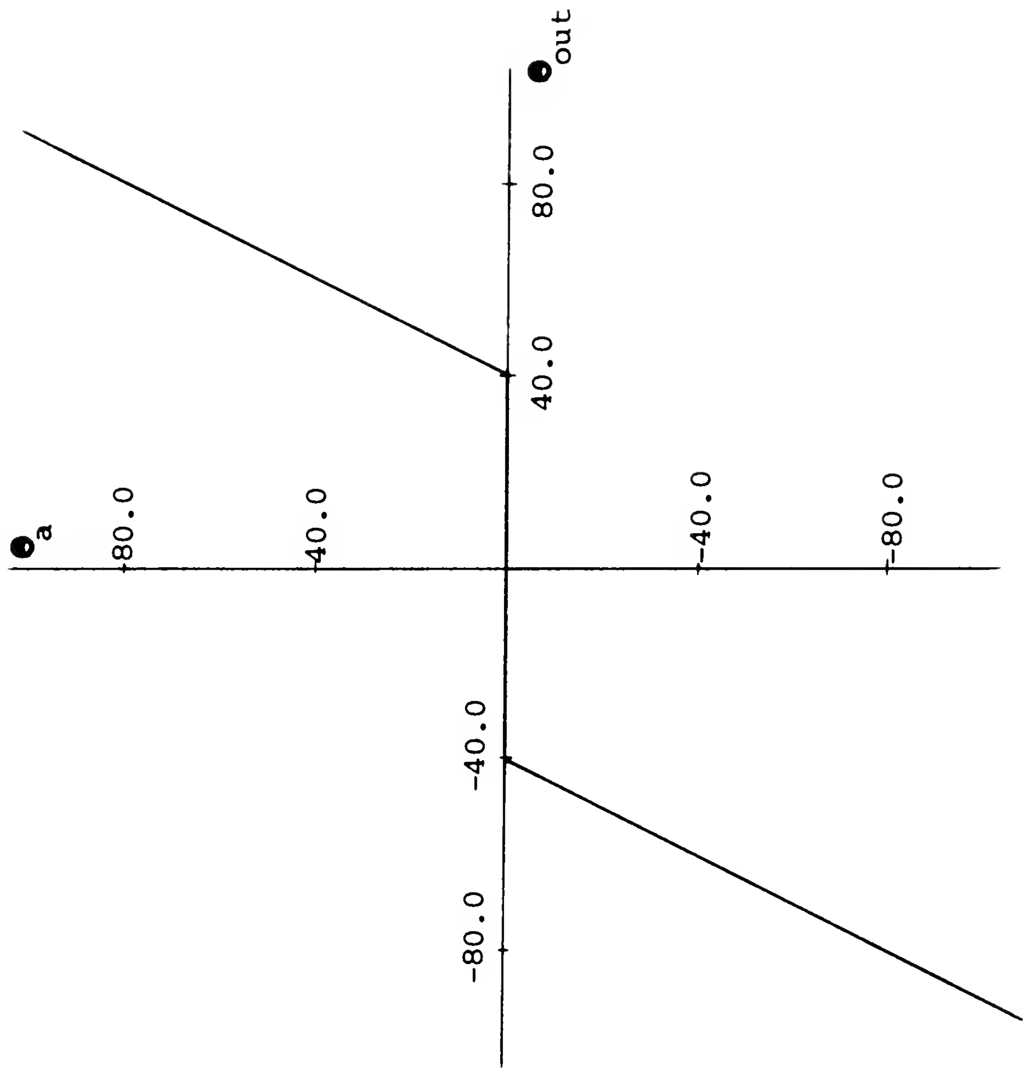

Figure 4.6 Dead zone element of second example

Substituting equation (4.14) into equation (4.12),

$$\beta = \frac{2\Theta_{out} - 80}{\Theta_{out}} \qquad \Theta_{out} \geq 40. \qquad (4.13b)$$

Dividing both numerator and denominator of the right-hand sides of equations (4.13a) and (4.13b) by Θ_{in},

$$\beta = 0.0 \qquad \frac{\Theta_{out}}{\Theta_{in}} \leq \frac{40}{\Theta_{in}} \qquad (4.15a)$$

$$\beta = \frac{\frac{2\Theta_{out}}{\Theta_{in}} - \frac{80}{\Theta_{in}}}{\Theta_{out}/\Theta_{in}} \qquad \frac{\Theta_{out}}{\Theta_{in}} \geq \frac{40.0}{\Theta_{in}} \qquad (4.15b)$$

Equations (4.15a) and (4.15b) can be solved for each Θ_{in}. For $\Theta_{in} = 10.0$,

$$\beta = 0.0 \qquad \frac{\Theta_{out}}{\Theta_{in}} \leq 4.0 \qquad (4.16a)$$

$$\beta = \frac{\frac{2\Theta_{out}}{\Theta_{in}} - 8.0}{\frac{\Theta_{out}}{\Theta_{in}}} \qquad \frac{\Theta_{out}}{\Theta_{in}} \geq 4.0 \qquad (4.16b)$$

For $\Theta_{in} = 20.0$,

$$\beta = 0.0 \qquad \frac{\Theta_{out}}{\Theta_{in}} \leq 2.0 \qquad (4.17a)$$

$$\beta = \frac{\dfrac{2\Theta_{out}}{\Theta_{in}} - 4.0}{\dfrac{\Theta_{out}}{\Theta_{in}}} \qquad \frac{\Theta_{out}}{\Theta_{in}} \geq 2.0 \qquad (4.17b)$$

For $\Theta_{in} = 40.0$,

$$\beta = 0.0 \qquad \frac{\Theta_{out}}{\Theta_{in}} \leq 1.0 \qquad (4.18a)$$

$$\beta = \frac{\dfrac{2\Theta_{out}}{\Theta_{in}} - 2.0}{\dfrac{\Theta_{out}}{\Theta_{in}}} \qquad \frac{\Theta_{out}}{\Theta_{in}} \geq 1.0 \qquad (4.18b)$$

For $\Theta_{in} = 80.0$,

$$\beta = 0.0 \qquad \frac{\Theta_{out}}{\Theta_{in}} \leq 0.5 \qquad (4.19a)$$

$$\beta = \frac{\dfrac{2\Theta_{out}}{\Theta_{in}} - 1.0}{\dfrac{\Theta_{out}}{\Theta_{in}}} \qquad \frac{\Theta_{out}}{\Theta_{in}} \geq 0.5 \qquad (4.19b)$$

Equations (4.16a), (4.16b), (4.17a), (4.17b), (4.18a), (4.18b), (4.19a) and (4.19b) give the relation between $\frac{\Theta_{out}}{\Theta_{in}}$ and β for the various constant Θ_{in} . This relation is made into table (IV.2) where $\frac{\Theta_{out}}{\Theta_{in}}$ is the enterring value and β is read out for a constant Θ_{in} .

β

$\frac{\Theta_{out}}{\Theta_{in}}$	$\Theta_{in} = 10.$	$\Theta_{in} = 20.$	$\Theta_{in} = 40.$	$\Theta_{in} = 80.$
0.0	0.0	0.0	0.0	0.0
0.5	0.0	0.0	0.0	0.0
1.0	0.0	0.0	0.0	1.0
1.5	0.0	0.0	0.667	1.33
2.0	0.0	0.0	1.00	1.5
2.5	0.0	0.4	1.20	1.6
3.0	0.0	0.667	1.33	
3.5	0.0	0.855	1.43	
4.0	0.0	1.00	1.5	
4.5	0.22	1.12	1.6	
5.0	0.4	1.2		
5.5	0.546	1.27		
6.0	0.667	1.33		
6.5	0.77	1.38		
7.0	0.856	1.43		
7.5	0.93	1.47		
8.0	1.00	1.50		
8.5	1.06	1.53		
9.0	1.11	1.56		
9.5	1.16	1.58		
10.0	1.20	1.60		
10.5	1.24			
11.0	1.27			
11.5	1.30			
12.0	1.33			

Table IV.2 β as a Function of $\frac{\Theta_{out}}{\Theta_{in}}$ and Θ_{in} for the Dead Zone

Using the PARAMS-5 subprogram of the PARAMS program, a plot of β versus $\frac{\Theta_{out}}{\Theta_{in}}$ with constant ω curves is obtained, as shown in figure (4.7). Using the relation between $\frac{\Theta_{out}}{\Theta_{in}}$ and β, tabulated in table (IV.2), constant Θ_{in} curves are drawn on figure (4.7). The intersection of the constant Θ_{in} curves and the ω curves in figure (4.7) gives the relation between frequency and β.

The last step in the analysis, is the drawing of the closed loop frequency response for the various inputs. Using the PARAMS-7 subprogram of the PARAMS program, a plot of $\frac{\Theta_{out}}{\Theta_{in}}$ (dB) versus frequency with constant β curves is obtained. This is figure (4.8). Using the relation between β and ω just found, the constant Θ_{in} curves are drawn on figure (4.8).

Figure (4.8) shows the effect of the various inputs. As Θ_{in} is decreased, the magnitude of the flat part of the response is decreased. At the same time, a jump resonance effect takes place. This jump resonance is opposite to the one found for the saturation problem of this chapter.

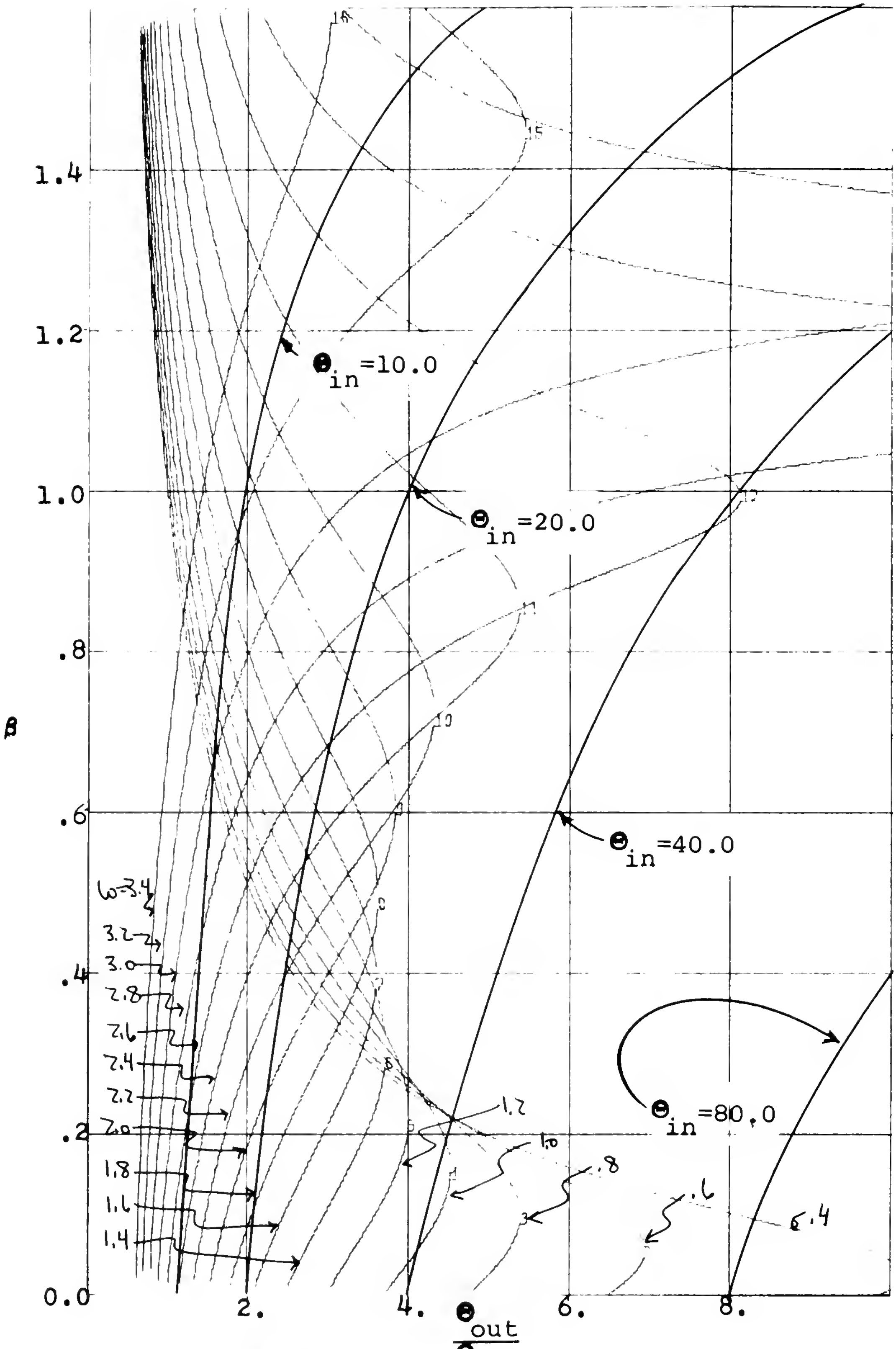

Figure 4.7 β versus $\frac{\Theta_{out}}{\Theta_{in}}$ magnitude for second example

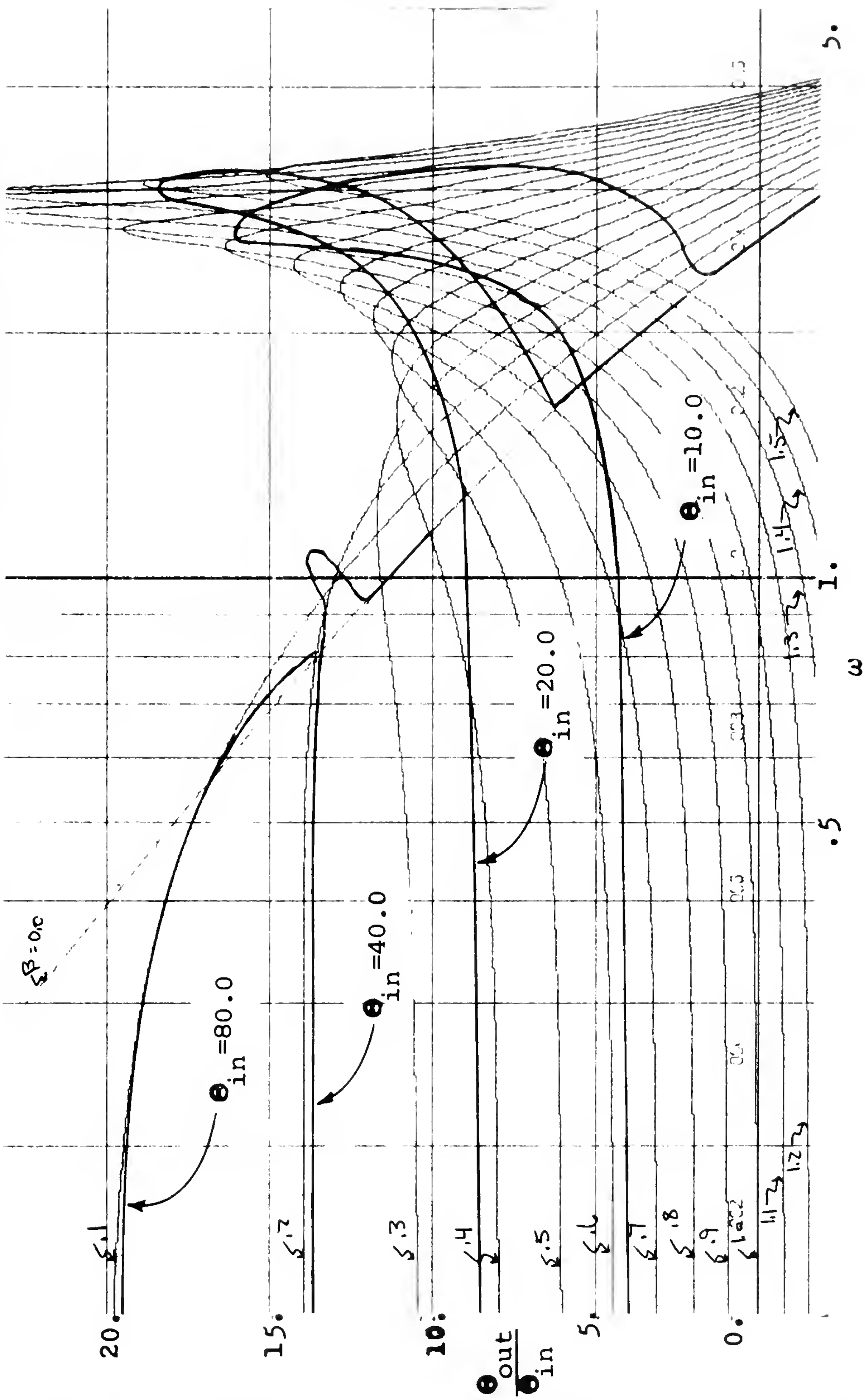

Figure 4.8 Closed loop frequency response for various θ_{in} in a system with dead zone

V. DESIGN TECHNIQUES

A. FIRST DESIGN PROBLEM

The previous chapters explored the techniques of analysis and showed that they worked for every example attempted. The question is whether or not these techniques can be reversed. The problem is to start with a given linear system, and design an element, linear or nonlinear, so that the system meets a certain specified response. It must be kept in mind that if a nonlinear element is used, it must be a signle-valued nonlinearity. To see whether or not this design can be accomplished, an example will be worked.

The design problem is a simple third order type 1 feedback system, where the closed loop transfer function is,

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{K_1K_2}{s(s+p_1)(s+p_2)+K_1K_2} \tag{5.1}$$

as shown in figure (5.1).

Let $p_1 = 1$, $p_2 = 2$, and $K_1 = 1$.

The first part of the problem is to find a K_2 so that the system is at the stability limit. This is easily done by conventional methods, and it is found that $K_2 = 6$. Equation (5.1) now becomes,

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{6}{s^3 + 3s^2 + 2s + 6} \tag{5.2}$$

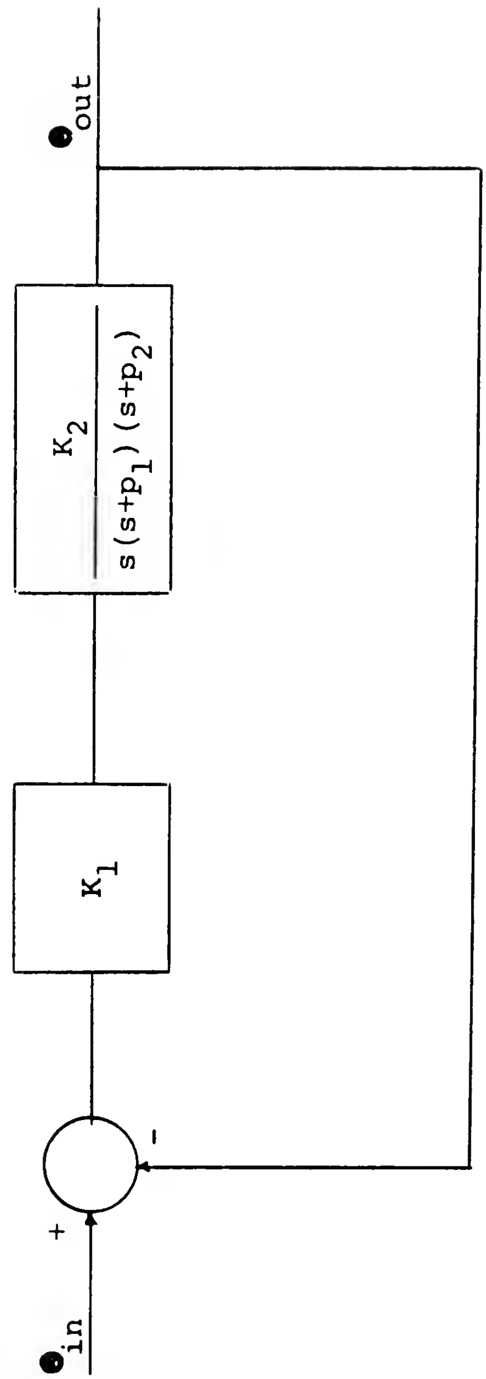

Figure 5.1 Type 1 third order feedback system

The second part of the design problem is also linear. The requirement is to provide tachometer feedback in a minor loop so that $M_p(\omega) = 2.0$. The block diagram of the system now is figure (5.2).

As has been previously shown, the PARAMS program can solve this aspect of the problem.[4] The transfer function of the system is now,

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{6}{s^3 + 3s^2 + 2s + 6 + 6K_t s} \qquad (5.3)$$

where K_t is the gain of the tachometer feedback element.

The PARAM-5 subprogram of the PARAMS program provides an easy method for determining K_t when $M_p(\omega) = 2.0$. Figure (5.3) gives a plot of K_t, tachometer feedback gain, versus the magnitude of $\frac{\Theta_{out}}{\Theta_{in}}$. Constant ω curves have been drawn on the plot for the region of frequency that is of interest. A vertical line is drawn at $\frac{\Theta_{out}}{\Theta_{in}} = 2.0$. This line represents the desired $M_p(\omega)$. The intersection of any horizontal line with the constant ω curves gives the frequency response of the system. The ordinate value of this line is K_t. Thus the horizontal line at $K_t = 0.35$ gives the desired $M_p(\omega) = 2.0$.

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[4] Glavis, G.O., <u>Frequency Response in the Parameter Plane</u>, Master's Thesis, Naval Postgraduate School, 1967.

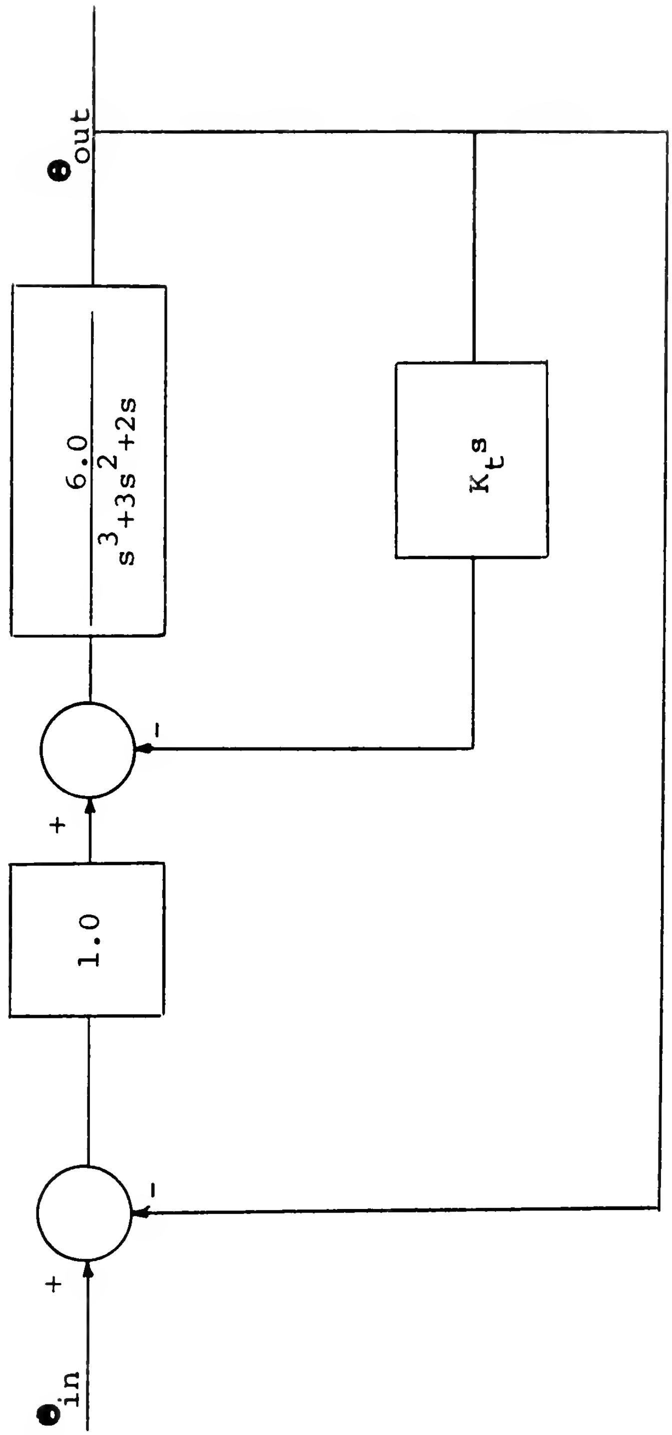

Figure 5.2 Third order feedback system with tackometer feedback

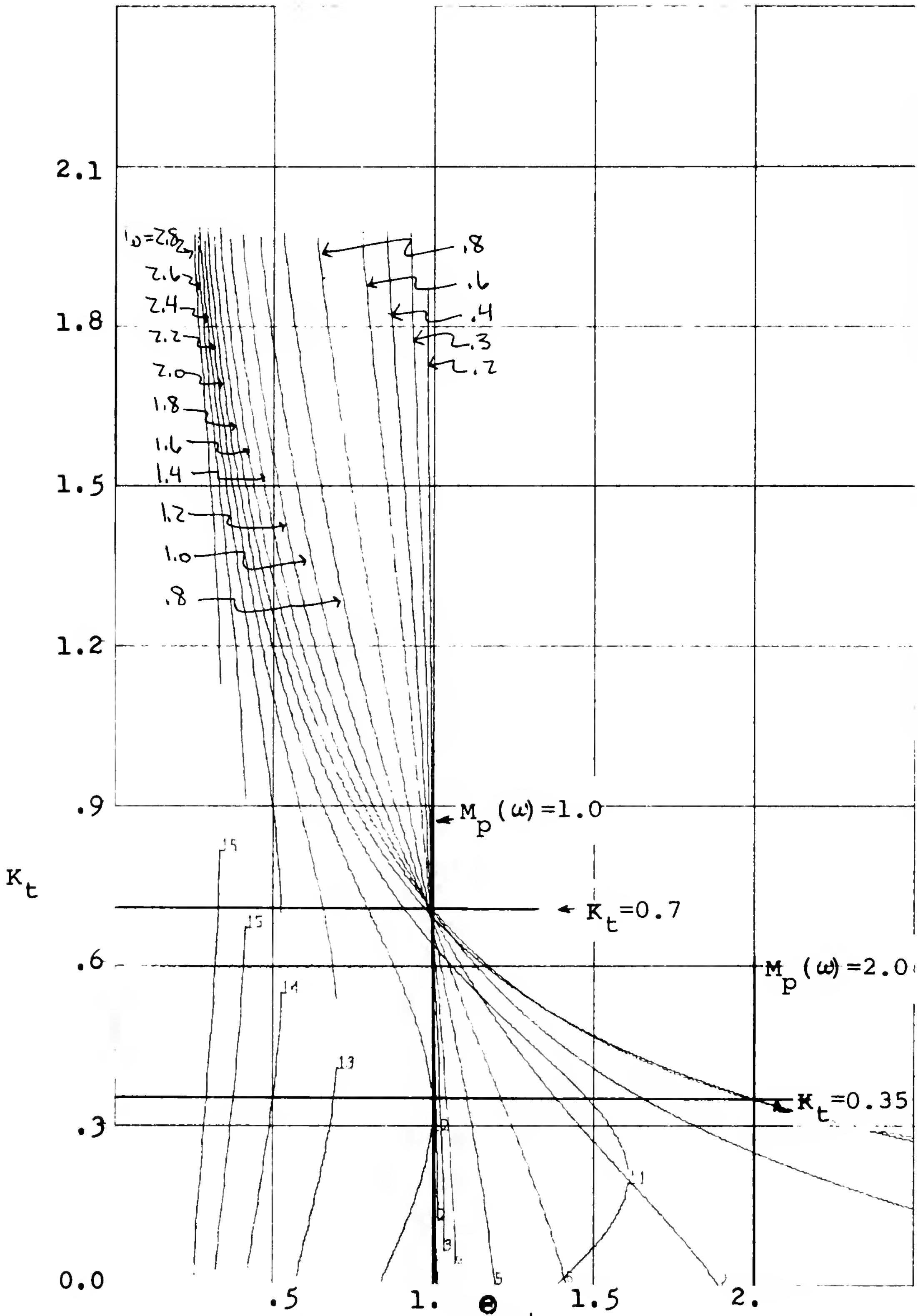

Figure 5.3 K_t versus $\frac{\Theta_{out}}{\Theta_{in}}$ magnitude for design problem

The last step in the design is to change $M_p(\omega)$ so that $M_p(\omega) = 1.2$. This is to be done by putting either a non-linear or linear element in the minor feedback path. This additional requirement changes the block diagram so that it now looks like figure (5.4), where β represents the gain of the nonlinear or linear element. Many techniquess can be used for designing a compensator to accomplish this new requirement. However in this particular case, a restriction will be placed on the system. The requirement is that the compensator be either a constant gain device, i.e., an amplifier or attenuator, or a variable gain device, such as a single-valued nonlinearity.

The next step in this design is similar to the previous step. A plot of nonlinear or linear gain (or attenuation) versus $\frac{\Theta_{out}}{\Theta_{in}}$ magnitude ($M_p(\omega)$) is to be made. This is done by the PARAM-5 subprogram of the PARAMS program. The plot is shown as figure (5.5). The ordinate is β, the linear or nonlinear gain, which ranges from 0.0 to 2.1. The abscissa is $\frac{\Theta_{out}}{\Theta_{in}}$ magnitude. A vertical line is drawn at $\frac{\Theta_{out}}{\Theta_{in}} = 1.2$. Because the requirements of the problem state that $M_p(\omega) = 1.2$, no design solution can extend to the right of the $\frac{\Theta_{out}}{\Theta_{in}} = 1.2$ line in figure (5.5). This region is shaded, and is a 'restricted design zone' as far as the design is concerned.

The final result in this design is to develop the shape of an element which, when put in the system for β in figure

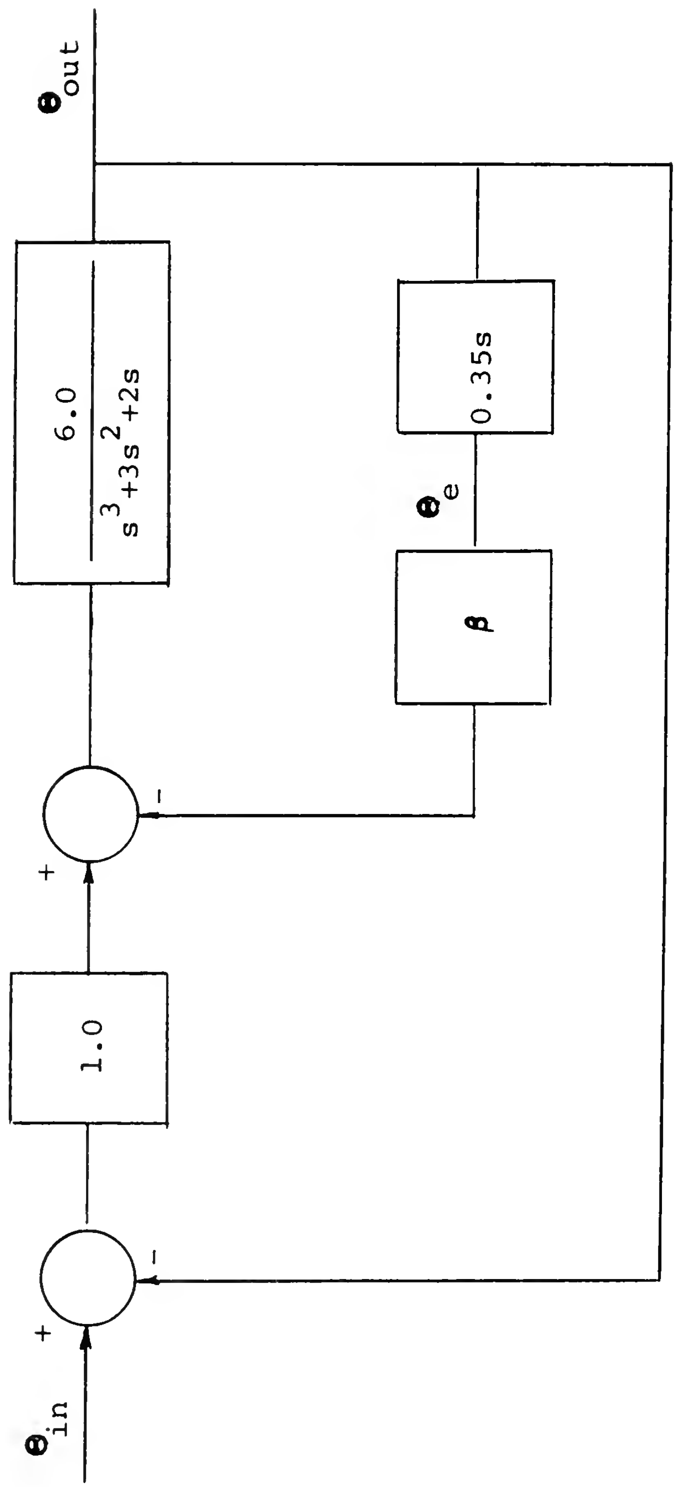

Figure 5.4 Third order system with linear/nonlinear compensator in tackometer feedback loop

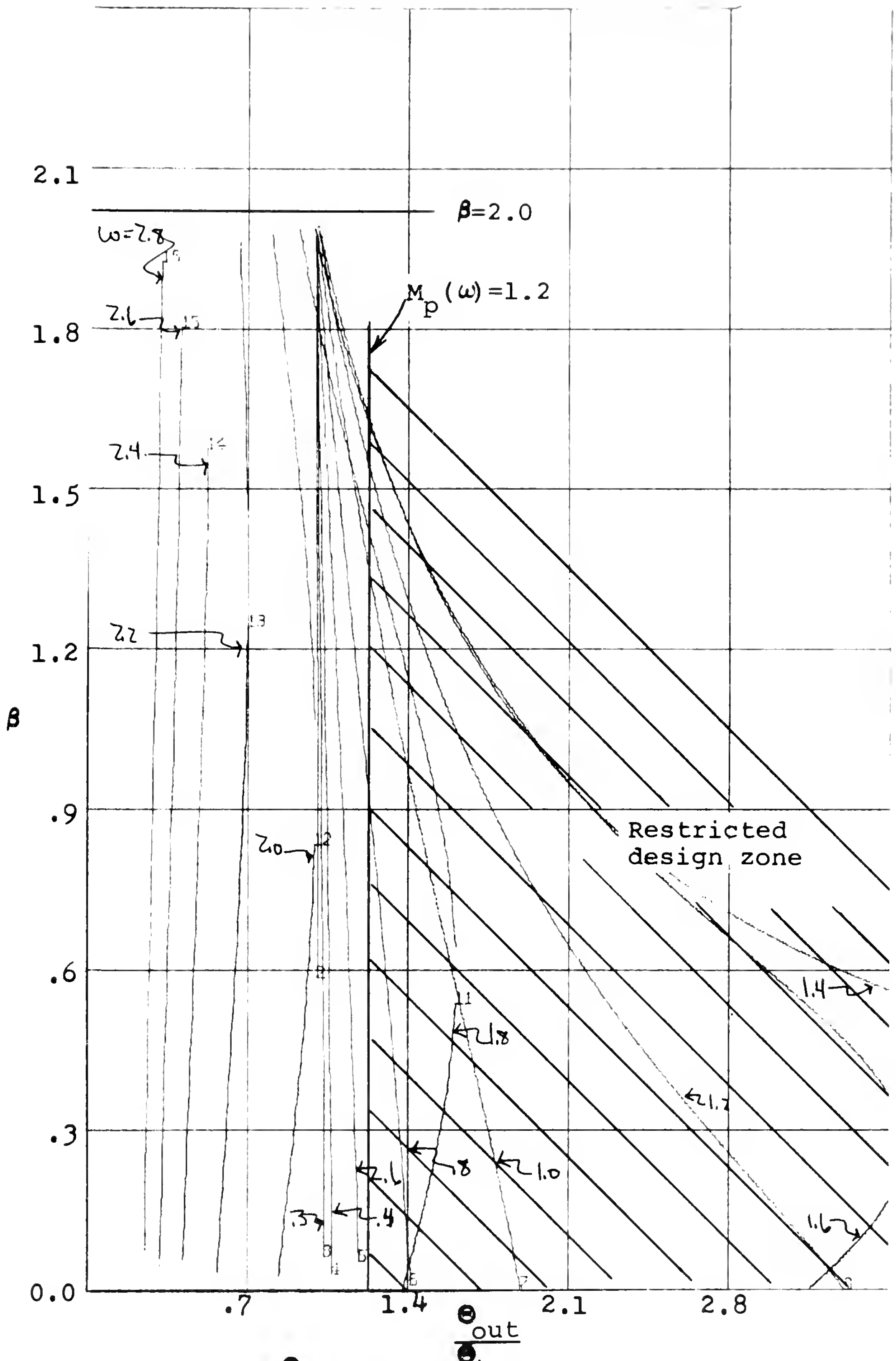

Figure 5.5 β versus $\frac{\Theta_{out}}{\Theta_{in}}$ magnitude for design problem

(5.4), gives the desired response. Before this can be obtained, steps similar to those in chapters 2,3 and 4 must be followed. These steps, however, must be reversed. Having determined in figure (5.5) the region in which design cannot take place, a relationship between this region and the variable β is found. The relationship has the coordinates of β and frequency. The next step is to find a relationship between β and the signal going into the linear or nonlinear element, using the relationship between β and ω.

It is difficult to find a relationship between a variable gain, β, and Θ_e in terms of frequency. However, it is a relatively simple process to find a relationship between β and $\frac{\Theta_e}{\Theta_{in}}$ in terms of frequency. Looking at figure (5.4),

$$\frac{\Theta_e}{\Theta_{out}} = K_t s = 0.35s \tag{5.4}$$

Multiplying both sides of equation (5.4) by Θ_{out},

$$\Theta_e = .35s\ \Theta_{out} \tag{5.5}$$

And dividing both sides of equation (5.5) by Θ_{in},

$$\frac{\Theta_e}{\Theta_{in}} = 0.35s\ \frac{\Theta_{out}}{\Theta_{in}} \tag{5.6}$$

By substituting $K_t = 0.35$ into equation (5.3) and letting the nonlinear or linear element be represented by β, equation (5.3) becomes,

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{6}{s^3 + 3s^2 + 2s + 6 + 2.1\beta s} \tag{5.7}$$

Substituting equation (5.7) into equation (5.6),

$$\frac{\Theta_{e}}{\Theta_{in}} = \frac{2.1s}{s^3 + 3s^2 + 2s + 6 + 2.1\beta s} \tag{5.8}$$

Equation (5.8) is used in the PARAM-5 subprogram to produce figure (5.6). Figure (5.6) has β, the variable gain, as the ordinate and $\frac{\Theta_e}{\Theta_{in}}$ magnitude as the abscissa. The same constant ω curves that are on figure (5.5) are on figure (5.6). Using β and ω as the coordinates for the restricted design zone in figure (5.5), this zone can be transferred to figure (5.6). This restricted design zone represents the area in which design cannot take place. The line that is the left-hand boundary of the zone is the critical boundary which will be used to formulate a non-linear element.

Referring to figure (5.4) again, it is seen that,

$$\beta = \frac{\Theta_a}{\Theta_e} \tag{5.9}$$

And that,

$$\Theta_a = \beta\,\Theta_e \tag{5.10}$$

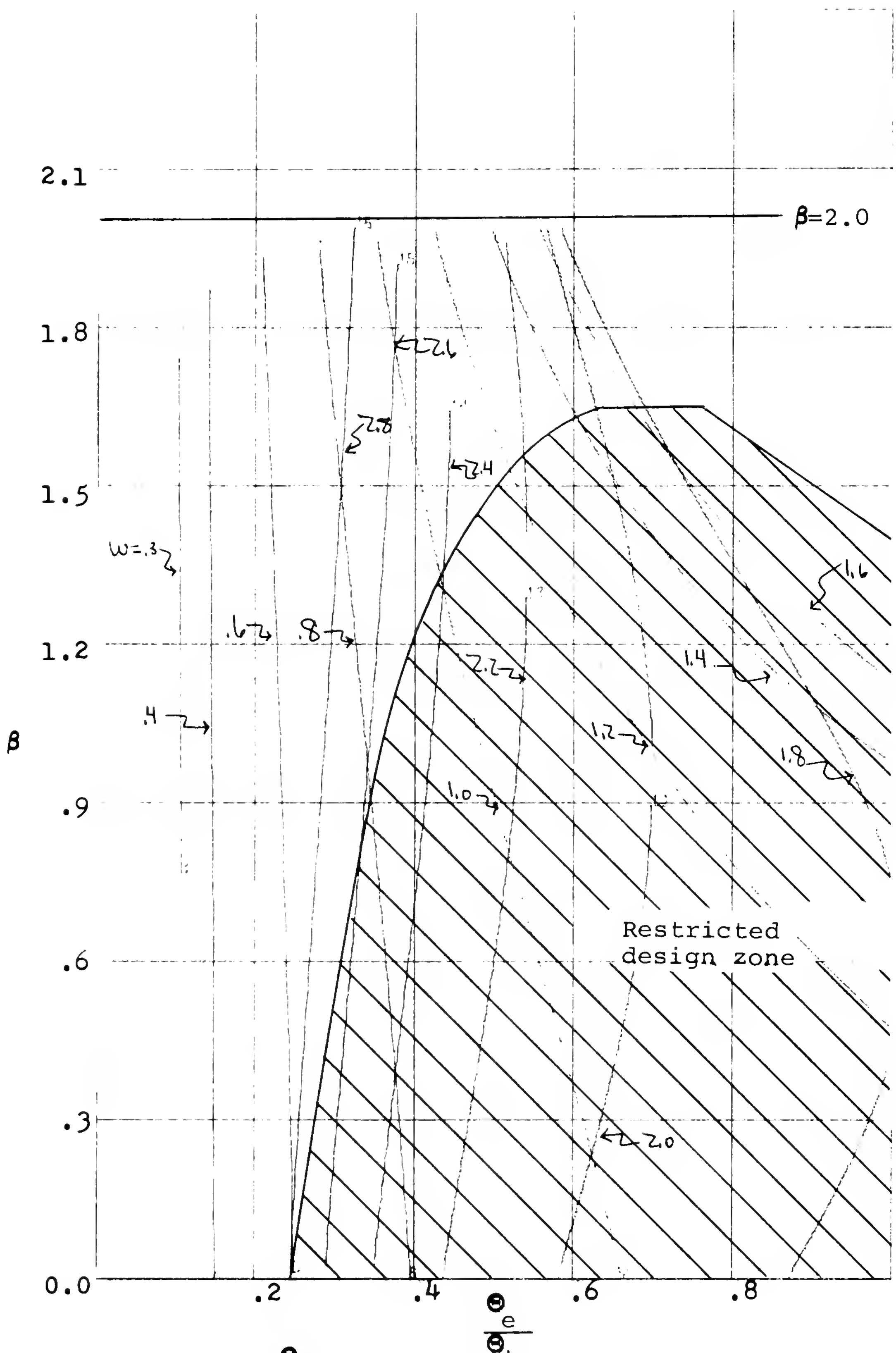

Figure 5.6 β versus $\frac{\Theta_e}{\Theta_{in}}$ magnitude for the design problem

Recalling that Θ_{in} is assumed a constant, the abscissa of figure (5.6) is Θ_e when $\Theta_{in} = 1.0$. From equation (5.5) and figure (5.6), the restricted design zone with its left-hand boundary line can be reconstructed in a plot of Θ_a versus Θ_e, where Θ_a is the signal out of the design element. This is depicted in figure (5.7) and provides a ready means of design.

When looking at figure (5.7), a simple solution to the problem of making $M_p(\omega) \leq 1.2$ is seen immediately. The solution is to provide a constant gain of 2.0, or $\beta = 2.0$. This constant gain is shown as a straight line in figure (5.7). This solution could have been seen in figure (5.6) by having a horizontal line at $\beta = 2.0$. The $\beta = 2.0$ line in figure (5.6) does not pass through the restricted zone. This solution is also seen in figure (5.5) as a $\beta = 2.0$ line also. One could go back in the design process to where K_t was being determined, from figure (5.3), it is seen that by making $K_t = .7$, a $M_p(\omega) = 1.0$ is obtained. By making $\beta = 2.0$, one is making a new K_t, i.e., $K_t\text{new} = \beta K_t$ for a constant β.

However, looking at figure (5.7) again, it is seen that a nonlinear design is also possible. One such nonlinear design is to have a relay with dead zone. This relay is drawn on figure (5.7). For signals less than 0.25, the output is 0.0. For signals greater than 0.25, the

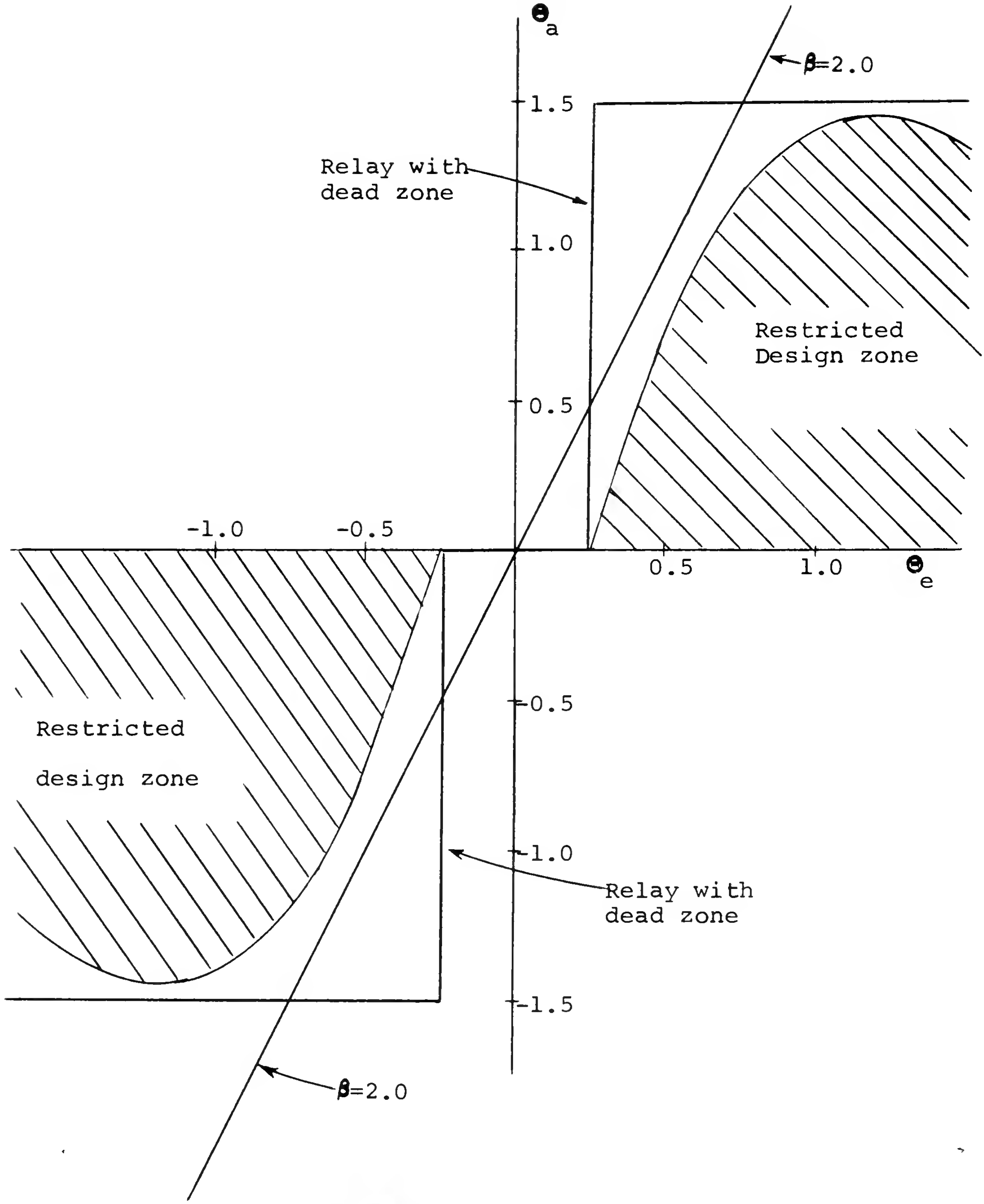

Figure 5.7 Plot of Θ_a-signal out of the element, versus Θ_e-signal into the element, with restricted design zone and proposed designs

output is 1.5. The relay with dead zone describing curve (constant Θ_{in} curve) can be redrawn in the following manner:

$$\beta = \frac{\Theta_a}{\Theta_e} \tag{5.11}$$

which comes from figure (5.4). For this relay with dead zone,

$$\Theta_a = 0.0 \qquad \Theta_e \leq 0.25 \tag{5.12a}$$

$$\Theta_a = 1.5 \qquad \Theta_e \geq 0.25 \tag{5.12b}$$

Substituting equation (5.12) into equation (5.11),

$$\beta = \frac{0.0}{\Theta_e} \qquad \Theta_e \leq 0.25 \tag{5.13a}$$

$$\beta = \frac{1.5}{\Theta_e} \qquad \Theta_e \geq 0.25 \tag{5.13b}$$

And dividing both the numerator and denominator of the right-hand side of equation (5.13) by Θ_{in}, and letting $\Theta_{in} = 1.0$ for part of the numerator,

$$\beta = 0.0 \qquad \frac{\Theta_e}{\Theta_{in}} \leq 0.25 \tag{5.14a}$$

$$\beta = \frac{1.5}{\Theta_e/\Theta_{in}} \qquad \frac{\Theta_e}{\Theta_{in}} \geq 0.25 \tag{5.14b}$$

From equation (5.14), a table may be constructed similar to those in the previous chapters. The entering value is $\frac{\Theta_e}{\Theta_{in}}$, and the value read off is β, the nonlinear variable gain. $\frac{\Theta_e}{\Theta_{in}}$ is the abscissa of figure (5.6) and β is the ordinate of figure (5.6). The result is table (V.1).

With table (V.1), the constant Θ_{in} curve for the relay can be reconstructed on figure (5.6). However, since figure (5.6) does not cover the range of β and $\frac{\Theta_e}{\Theta_{in}}$ needed, a new plot is obtained. This plot is figure (5.8) where β goes from 0.0 to 6.0 and $\frac{\Theta_e}{\Theta_{in}}$ goes from 0.0 to 2.4. It is the same plot as figure (5.6), except that the scales have been expanded. The restricted design zone is reconstructed on figure (5.8). Now the constant Θ_{in} curve for the relay with dead zone is plotted on figure (5.8). The intersection of the constant Θ_{in} curve and the ω curves gives the relationship between β and ω. Using this relationship in figure (5.8), the closed loop frequency response can be reconstructed to show that the relay does meet the specifications.

The PARAM-7 subprogram of the PARAMS program is used to reconstruct the closed loop frequency response. This plot is figure (5.9) and has $\frac{\Theta_{out}}{\Theta_{in}}$ magnitude (dB) versus ω, with constant β curves of β = 0.0, 0.5, 0.75, 1.0, 1.25, 1.5, 1.75, 2.0, 2.25, 2.5, 2.75, 3.0, 3.5, 4.0, 5.0 and 6.0 drawn on it. The frequency response for the relay is drawn on figure (5.9), using the coordinates of β and ω

$\frac{\Theta_e}{\Theta_{in}}$	β
0.0	0.0
0.1	0.0
0.2	0.0
0.25	0.0
0.25	6.0
0.3	5.0
0.35	4.28
0.4	3.75
0.45	3.33
0.5	3.00
0.55	2.73
0.6	2.5
0.65	2.31
0.7	2.14
0.75	2.00
0.8	1.87
0.85	1.77
0.9	1.69
0.95	1.58
1.0	1.50
1.1	1.36
1.2	1.25
1.3	1.15
1.4	1.07
1.5	1.00
1.6	0.93
1.7	0.88
1.8	0.83
1.9	0.79
2.0	0.75

Table V.1 β as a Function of $\frac{\Theta_e}{\Theta_{in}}$ for the Relay with Dead Zone.

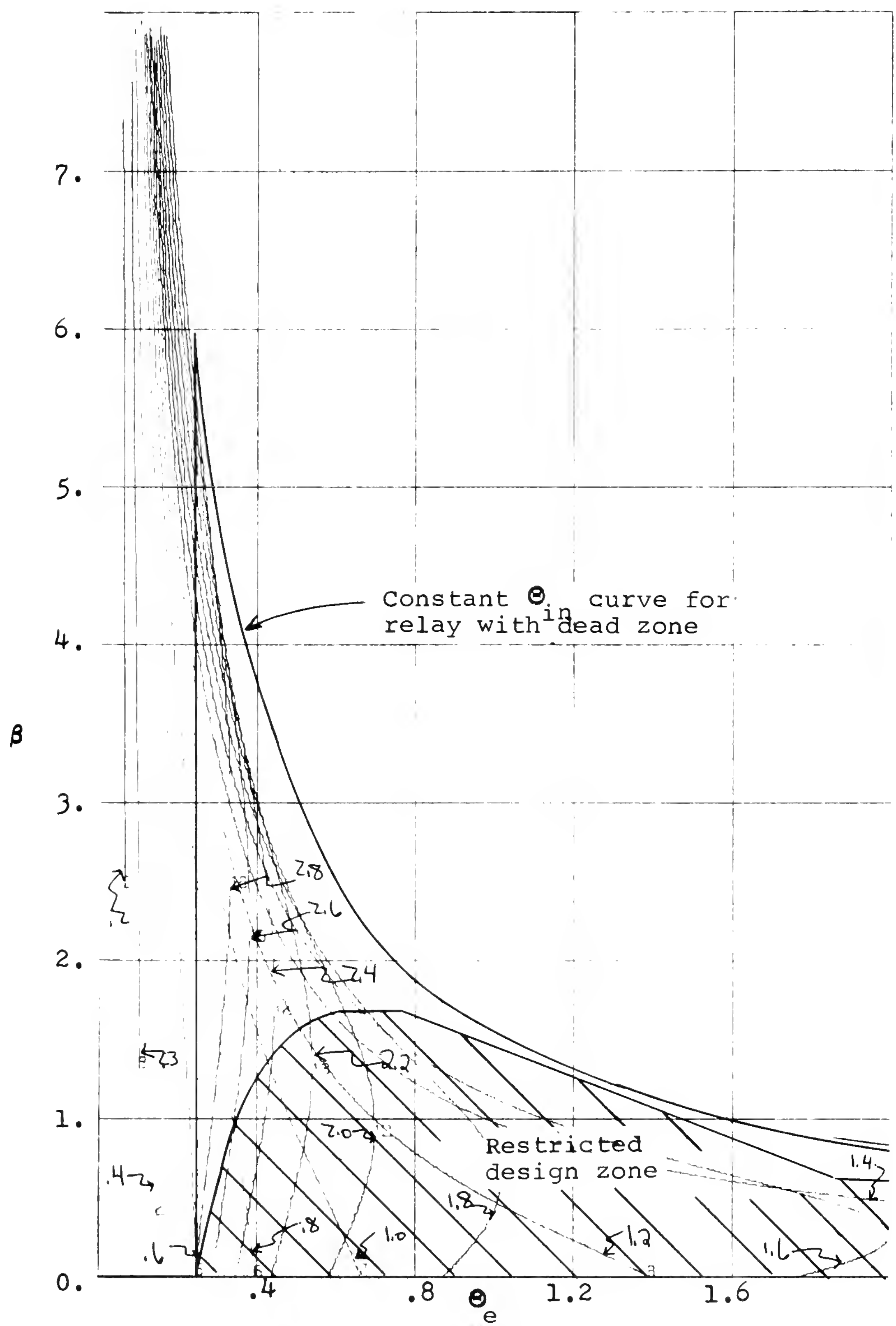

Figure 5.8 β versus $\frac{\Theta_e}{\Theta_{in}}$ magnitude with proposed relay with dead zone constant Θ_{in} curve

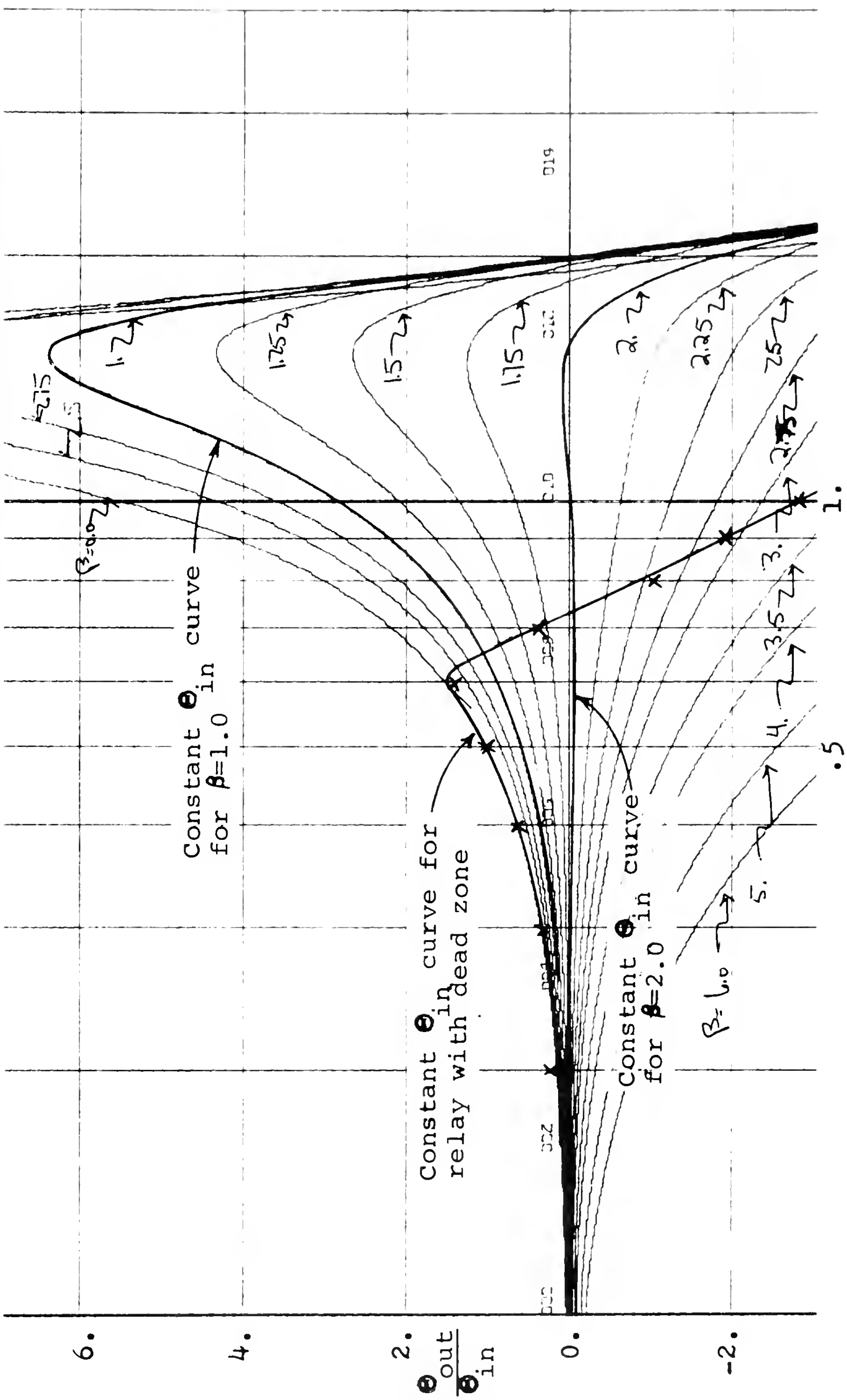

Figure 5.9 Closed loop frequency response of design problem with proposed design solutions

from figure (5.8). It is seen in figure (5.9) that the system with the relay does satisfy the conditions that $M_p(\omega) \leq 1.2$ (1.58 dB). Also drawn on figure (5.9) is the linear design solution, $\beta = 2.0$, which also satisfies the design conditions. A $\beta = 1$ curve is drawn on figure (5.8) to show the response of the system after determining K_t but before the design of the linear/nonlinear element. The solution has been checked by analog simulation and the results are plotted on figure (5.9) by "x's". The analog solution checks closely with the graphical design.

The steps in the design are summarized below. According to the problem, some steps may be eliminated while other steps may have to be added to solve the particular problem.

1. After all linear design and compensation have been completed, with only the nonlinear design left, obtain a plot of β, the variable gain of the nonlinear element, versus $\frac{\Theta_{out}}{\Theta_{in}}$ the magnitude, with constant ω curves drawn on the plot. This can be accomplished by using the PARAM-5 subprogram of the PARAMS program or some other similar computer program. The plot corresponding to the example in this chapter is figure (5.5).

2. Having obtained the plot in step 1, draw on it the area in which design cannot take place (a restricted design zone). This zone is determined by the specifications of the problem and must be in terms of $\frac{\Theta_{out}}{\Theta_{in}}$ magnitude and frequency.

3. Obtain the transfer function of the signal going into the nonlinear/linear element with respect to the input signal into the system. In the problem of this chapter, this transfer function is $\frac{\Theta_e}{\Theta_{in}}$. Using the transfer function with β representing the variable gain, obtain a second plot of β versus this transfer function with the same ω curve as were drawn in the plot in step 1. This corresponds to figure (5.6) in this chapter.

4. Using the coordinates of β and ω from the plot in step 1, transfer the restricted design zone to the plot obtained in step 3. One now has a relationship for the restricted design zone in terms of β and $\frac{\Theta_e}{\Theta_{in}}$.

5. Because Θ_{in} must be specified as a constant to obtain the response of a system with a nonlinear element, the plot of step 3 is a relationship between β and Θ_e for the restricted design zone. From this relationship, a plot of Θ_a (the signal out of the linear/nonlinear element) versus Θ_e (the signal into the element) can be drawn. The restricted design zone is then drawn on this plot. This new plot corresponds to figure (5.7) in this chapter.

6. From this plot in step 5, any linear nonlinear element can be designed that does not fall into the restricted design zone. The nonlinear element must be a single-valued nonlinearity, however.

7. Make a table of β as a function of $\frac{\Theta_e}{\Theta_{in}}$ for the element that was designed. This is done by using the plot

in step 6, realizing that $\beta = \frac{\Theta_a}{\Theta_e}$ and that Θ_{in} is a constant. This corresponds to table (V.1) in this chapter.

8. Having obtained this table in step 7, plot the designed element constant Θ_{in} curve on the plot of step 5. The coordinates of the constant Θ_{in} curve are β and $\frac{\Theta_e}{\Theta_{in}}$ and come from the table in step 7. Having plotted the constant Θ_{in} curve on this plot, one now has the relationship between β and ω for the constant Θ_{in} curve. This relationship comes from the intersection of the designed element's constant Θ_{in} curve and the constant ω curves.

9. The final step is the reconstruction of the closed loop frequency response. The open loop frequency response can be obtained in the same manner, except by using the open loop transfer function rather than the closed loop transfer function. Obtain a plot of $\frac{\Theta_{out}}{\Theta_{in}}$ versus the frequency, with constant β curves. The values of β of the design element should determine what β curves are needed in this plot. Replot the constant Θ_{in} curve onto this plot using the coordinates of β and ω found in step 8. The constant Θ_{in} curve on this plot should satisfy the design specifications.

B. SECOND DESIGN PROBLEM

In the previous analysis and design examples, the nonlinearities were functions of the signal into the element. A second type of nonlinearity is one that is a direct function of frequency. The analysis techniques of this thesis can be used for this second type of nonlinear element. To illustrate how these techniques are used, a design problem is worked.

The block diagram of the second design problem is figure (5.10). This system is the same example as was used in chapter II. α represents the nonlinear element which will be used to change the frequency response. From figure (5.10), the closed loop transfer function is,

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{600\alpha}{s^4 + 18s^3 + 95s^2 + 150s + 600\alpha} \qquad (5.15)$$

Using the equation (5.15) and the PARAM-7 subprogram, a plot of $\frac{\Theta_{out}}{\Theta_{in}}$ (dB) versus ω is obtained. This plot is figure (5.11) and has constant α curves plotted on it. Also on figure (5.11) is the desired frequency response of the system. The desired frequency response can be described in terms of α and ω from figure (5.11).

From figure (5.10), $\frac{\Theta_e}{\Theta_{in}}$ is written,

$$\frac{\Theta_e}{\Theta_{in}} = \frac{s^4 + 18s^3 + 95s^2 + 150s}{s^4 + 18s^3 + 95s^2 + 150s + 600\alpha} \qquad (5.16)$$

With equation (5.16) and the PARAM-5 subprogram, a plot of α versus $\frac{\Theta_e}{\Theta_{in}}$ is made. This plot is figure (5.12) and has constant ω curves drawn on it. Using the coordinates of α and ω, the desired frequency response can be

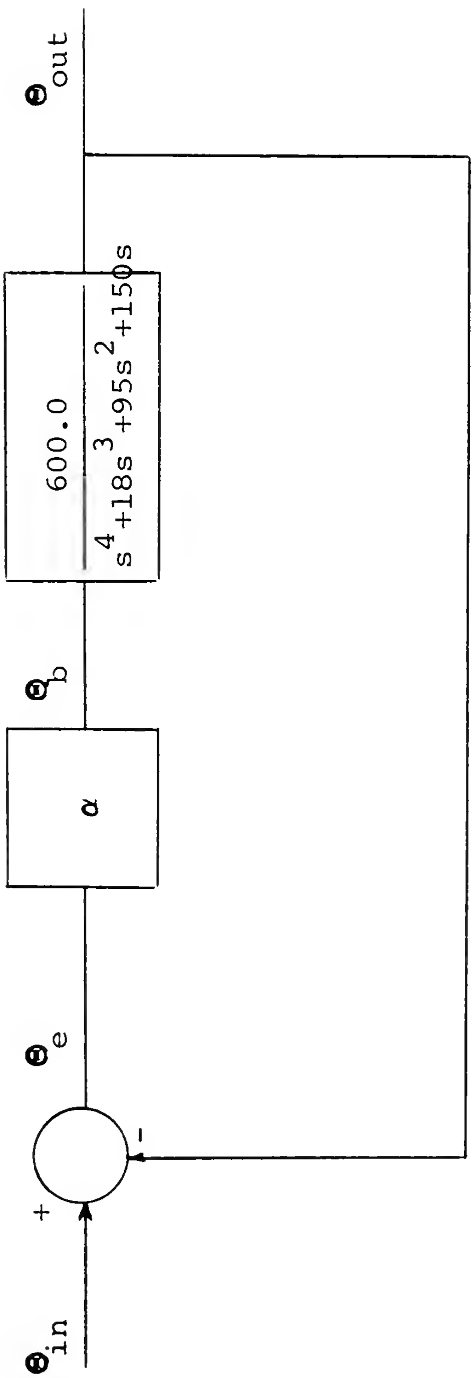

Figure 5.10 Block diagram of second design problem

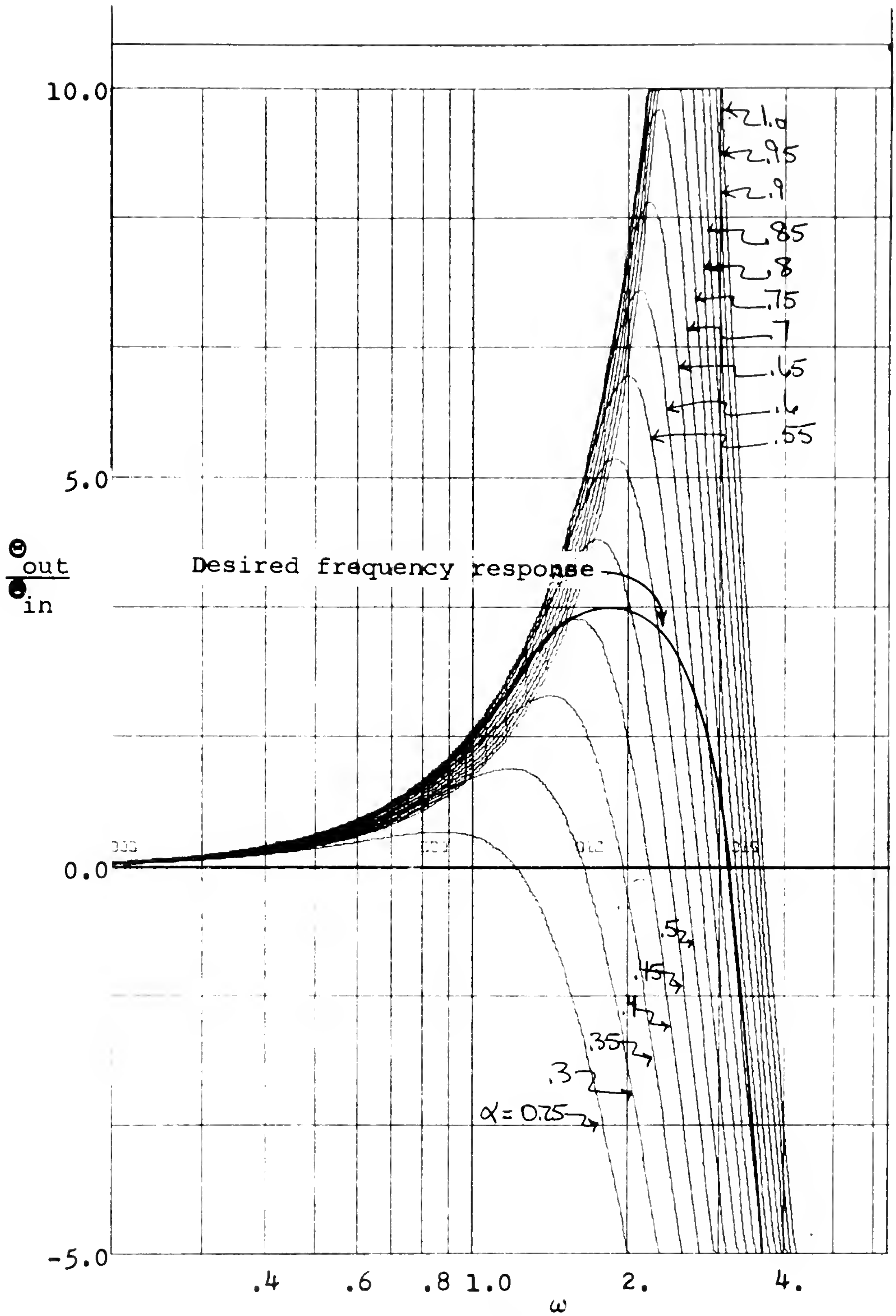

Figure 5.11 Desired closed loop frequency response of second design problem

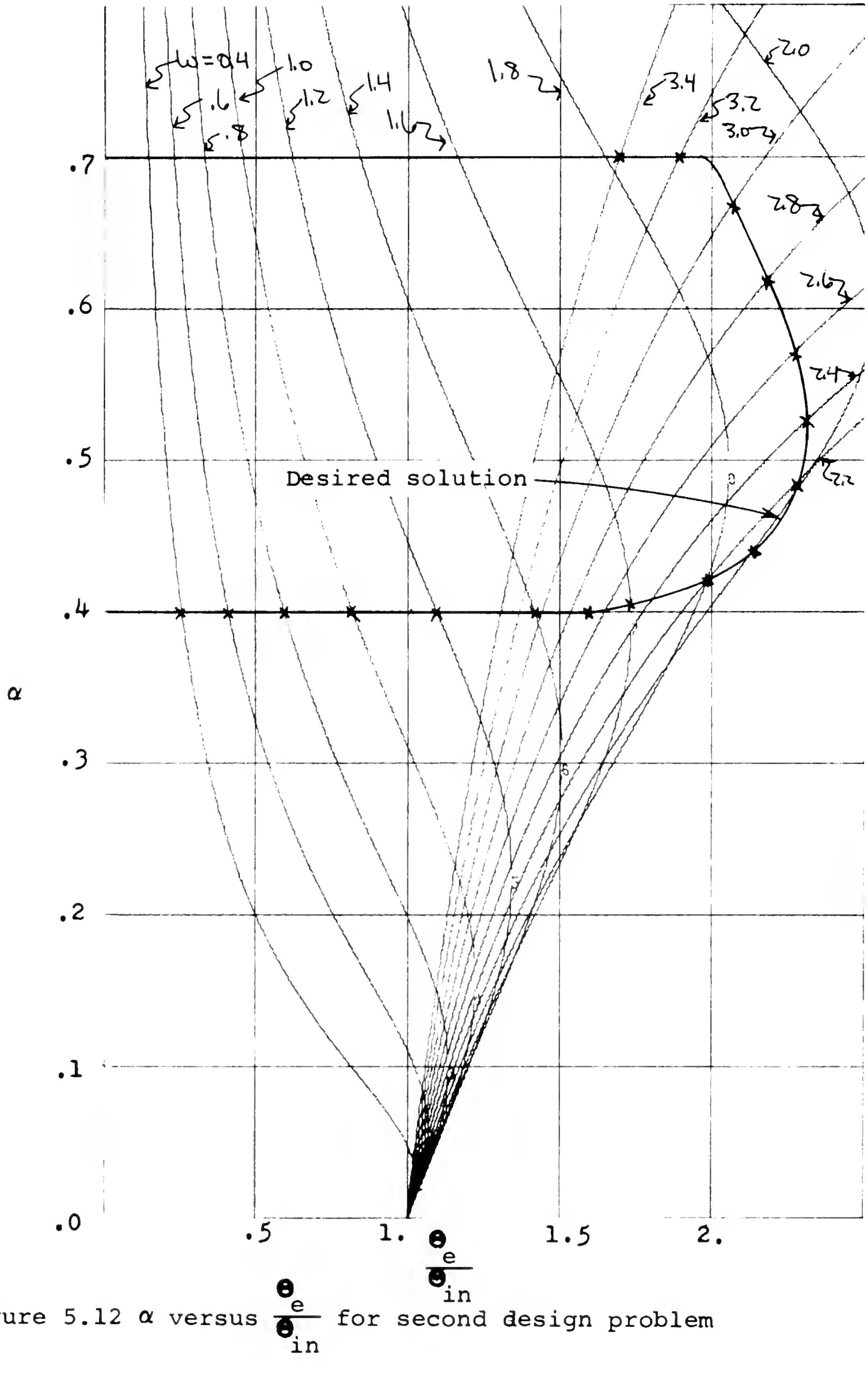

Figure 5.12 α versus $\frac{\theta_e}{\theta_{in}}$ for second design problem

replotted from figure (5.11) onto figure (5.12). As can be seen in figure (5.11), the desired solution is a function of frequency, not a function of $\frac{\Theta_e}{\Theta_{in}}$ as was the case in previous examples. If the solution were a function of $\frac{\Theta_e}{\Theta_{in}}$, then for a given frequency, there would be two values of gain in figure (5.12). This would not give the proper frequency response. Figure (5.12) gives the desired solution in terms of α and ω with $\frac{\Theta_e}{\Theta_{in}}$ as a third parameter. $\frac{\Theta_e}{\Theta_{in}}$ can only be used as a parameter when $\frac{\Theta_e}{\Theta_{in}}$ is given with an associated value of frequency.

It can be clearly seen from figure (5.12), that for $\omega \leq 1.5$, the gain, α, is 0.4. Between $\omega = 1.5$ and $\omega = 3.2$, the nonlinear device would have a gain varying bewteen 0.4 and 0.7. For $\omega \geq 3.2$, the gain of the element would be 0.7. This description of the nonlinear device can also be seen on an input-output plot of the nonlinear element. This diagram is figure (5.13) where a straight line segment represents the constant gain of the element.

The response of the system is not dependent on Θ_{in}. Thus, for this design solution, the frequency response is the same for all values of Θ_{in}. The realization of the desired nonlinear element could possibly consist of a variable amplifier with a frequency discriminator to determine what the gain of the element should be at a certain frequency.

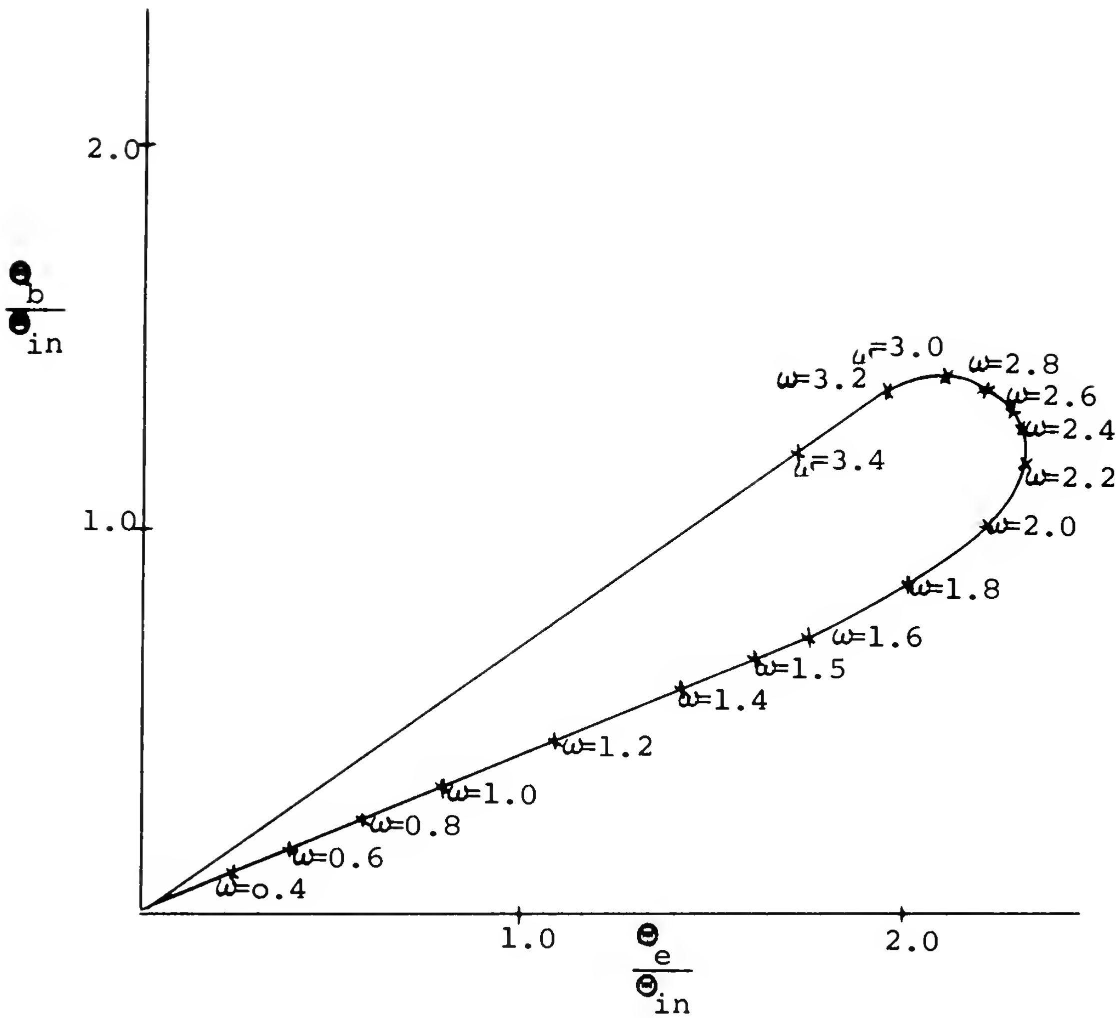

Figure 5.13 Characteristics of the nonlinear element of the second design problem

VI. DISCUSSION, CONCLUSIONS AND AREAS OF FUTURE STUDY

A. DISCUSSION

The accuracy of the solution of both the analysis and design depends on graphical accuracy. The plots made by the digital computer were accurate enough for the study. Any errors in the solutions came from plotting of the curves by hand and reading the data from the plots. In some cases where the scale of the plots was small and accuracy was critical, some small errors appeared in the solution.

All the examples, except for the open loop analysis, were checked by analog simulation. Appendix A explains the simulation. The nonlinear elements were simulated by using electronic switching and digital logic circuits. As a result, the nonlinear elements are ideal elements. In a real example, these elements may not be ideal, but instead show different characteristics. This presents no problem when using these analysis and design techniques as long as the nonlinear element remains single-valued. Any single-valued nonlinearity can be used in the analysis and design.

B. CONCLUSIONS

The main conclusion to be drawn from this study is that a workable, practical and reliable method has been developed to obtain the open and closed loop frequency response of systems with a single-valued nonlinearity. This includes both magnitude and phase response. The method is fast, accurate, gives a graphical solution and involves no laborious calculations.

The method of analysis can be reversed and be used for synthesis without complicating the problem. The design techniques show clearly what are the possible design solutions, linear and nonlinear, including their exact parameters. As in the analysis, the nonlinear elements are restricted to single-valued elements. As shown in the second design problem, the analysis techniques can also be reversed to handle design solutions that are a function of only the frequency of the system. The desired frequency response remains independent of the magnitude of the signal going into the system.

The primary problem was finding the relationship between the nonlinearity and the frequency of the system. This method provides a very simple technique which permits the nonlinearity to be expressed as a variable gain and relates this gain to the frequency of the system simply and accurately.

The analysis techniques can be used for predicting jump resonance. The prediction includes not only the fact that jump resonance will occur but also the frequency of the jump resonance. This prediction can be made in the first few steps of the analysis. The final frequency response shows the jump resonance in both magnitude and phase.

C. AREAS OF FUTURE STUDY

One of the basic areas that needs investigation is in the phase domain. With slight modification to the PARAMS program, phase can be added as one of the variables in plots other than just the PARAM-7 frequency response plot.

With the addition of phase, an extension of this method may be possible so that two-valued nonlinearities could be analyzed. These nonlinearities could include backlash, negative deficiency, two-position relay, etc. These types of nonlinearities could also possibly be studied by breaking them down into real and imaginary parts and using the two parameter capability of the PARAMS program to handle these two parts.

Another area left unexplored is analyzing the system with two single-valued nonlinearities. This could be developed as follows. The last step in the analysis technique was replotting the constant Θ_{in} curve to a plot of $\frac{\Theta_{out}}{\Theta_{in}}$ versus ω to obtain the frequency response. Instead of doing this replotting, transfer the constant Θ_{in} curve to a plot of the nonlinear gain versus $\frac{\Theta_{out}}{\Theta_{in}}$. In the PARAMS program, some element had to be designated as another parameter value and held constant for the analysis. By making a series of plots of nonlinear gain versus $\frac{\Theta_{out}}{\Theta_{in}}$

with the constant Θ_{in} curve, each plot having the second parameter set to a different value, a three dimensional surface of the constant Θ_{in} curve is developed. The coordinates would be α, the nonlinear gain, β, the second parameter, $\frac{\Theta_{out}}{\Theta_{in}}$ and ω. By repeating this with a second nonlinear element, a second constant Θ_{in} surface is obtained. The intersection of these two surfaces at a certain ω and $\frac{\Theta_{out}}{\Theta_{in}}$ gives the closed loop frequency response of the system with two nonlinear elements.

This technique just described would be of course very laborious. It could be greatly simplified by developing a digital computer program which would find the surfaces and their intersections. This could then be extended into a four dimensional space program to handle three nonlinearities, five dimensional space to handle four nonlinearities, etc.

APPENDIX A. ANALOG SIMULATION

The analog simulation was performed on the Donner 10/20 analog computer and two EAI TR-20 analog computers. The Donner 10/20 was used to simulate the linear portion of the system and the two EAI TR-20s were used to simulate the nonlinear element.

The simulation of the linear portion of a system was done by conventional analog simulation techniques. The main consideration was to preserve the position of the nonlinear element in the circuit so that no accidental manipulation occurred. This manipulation would correspond to incorrectly manipulating a block diagram with a nonlinear element. The reason for this care is the fact that the response of a system is a function of where the nonlinearity is placed in the system.

Scaling had to be used throughout the simulation and particularly in using the Donner with the two EAI computers. The Donner is a 100 v. machine and the EAIs are a 10 v. machines.

The success of the total simulation rested with the nonlinear element simulation. For this two EAI TR-20 electronic comparators were used, one electronic comparator in each machine. Each electronic comparator consists of a comparator unit which produces binary outputs and two electronic switching units. The comparator and switches have a switching time of 1.0μ sec. and the comparator has a switching sensitivity of ± 1.0μv. The block diagram of one electronic comparator is shown in figure (A.1).[5]

The electronic comparator can be patched so that it compares two inputs, X and Y, so that,

$$X + Y \geq 0.0 \qquad E_o = E_1 \tag{A.1a}$$

$$X + Y < 0.0 \qquad E_o = E_2 \tag{A.1b}$$

Where E_1 and E_2 are two arbitrary inputs and E_o is the output of the electronic comparator.

One electronic comparator was used for positive signals and the other electronic comparator was used for negative signals. Two examples are worked to show how the units were used.

The first example is an ideal relay (such a figure (3.18)) where it is desired that for all positive signals into the relay, the output is 4.0 volts. For all negative signals into the relay, the output is -4.0 volts. For the first electronic comparator, X is the input signal, Y is zero, E_1 is 4.0 volts and E_2 is the input signal. Thus, when $X + Y \geq 0.0$, which is when the input signal is greater than zero, the output from the first electronic comparator is E_1 or 4.0 volts. When $X + Y < 0.0$, which is

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[5] Electronic Associates, Inc. <u>TR-20 Computer Operator's Reference Handbook</u>, 1964.

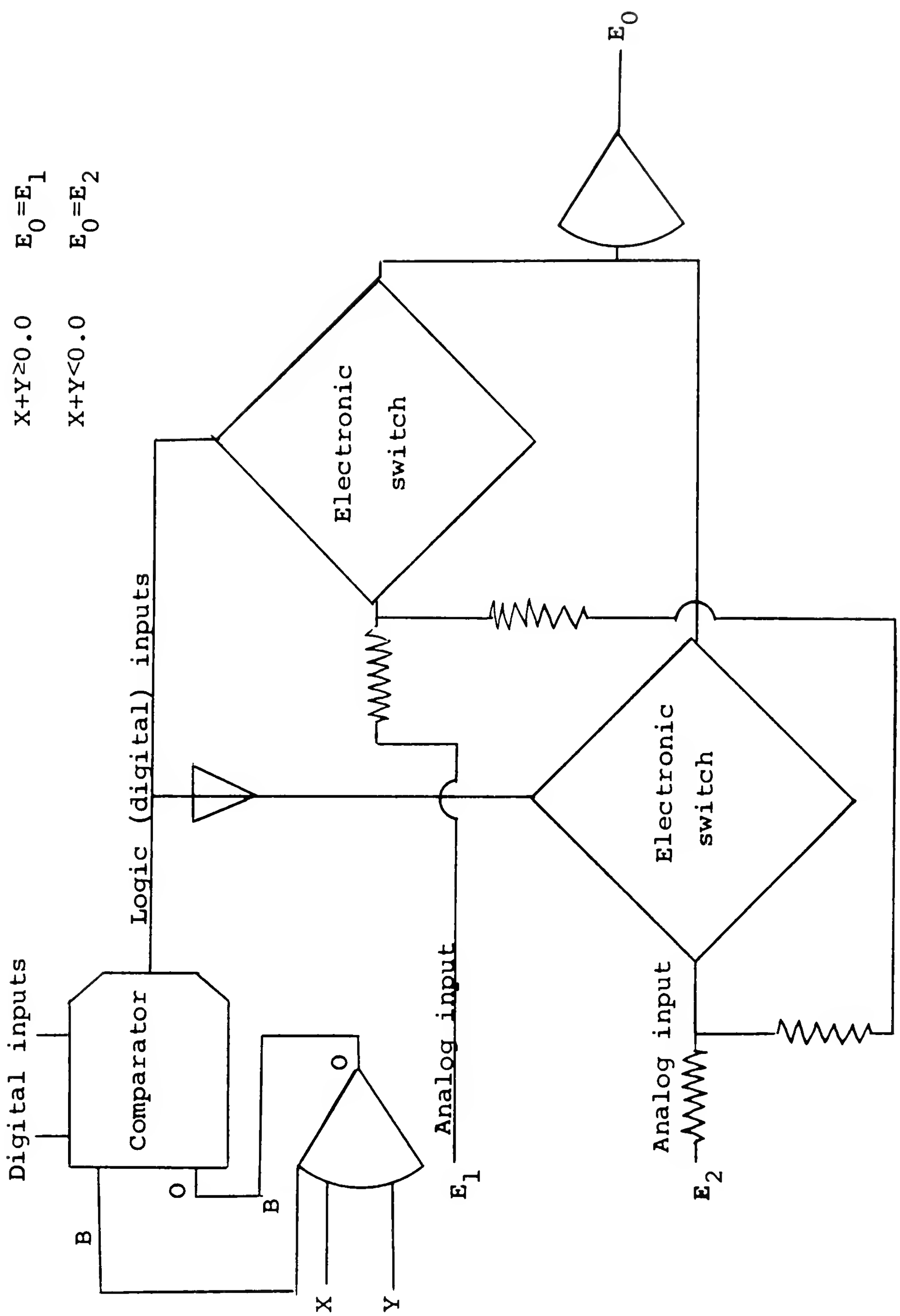

Figure A.1 EAI TR-20 electronic comparator when the input signal is negative, the output of the comparator is E_2 or the negative portion of the input signal. The output, E_o, of the first electronic comparator is made the X and the E_1 of the second electronic comparator. The Y is zero and E_2 is -4.0 volts. Thus when the output of the first electronic comparator is negative, the output of the second electronic comparator is -4.0 volts. When the output of the first electronic comparator is positive, the output of the second electronic comparator is +4.0 volts which is the only positive output of the first electronic comparator.

The second example is a dead zone element whose dead space extends ± 4.0 v. Let the input into the dead zone element be represented by the symbol IN. For the first electronic comparator X = IN, Y = 4.0 v., E_1 = IN - 4. and E_2 = 0.0. Thus for positive signals greater than +4.0 v., the output, E_{01}, of the first electronic comparator is IN-4. For signals less than +4., E_{01} = 0.0. For the second electronic comparator let X = IN, Y = +4., $E_1 = E_{01}$ and E_2 = IN + 4. Thus for signals less than -4., the output, E_{02}, is IN + 4. For signals greater than -4., $E_{02} = E_{01}$ which is zero for $-4.0 \leq IN \leq 4.0$ and IN - 4. for IN > 4.

With the switching time of 1.0μsec. and sensitivity of 1μv., the response is very good. The output of the second electronic comparator is fed back into the linear system with appropriate scaling.

```
C******* PROGRAM PARAMS **********                                        0003
C  THIS PROGRAM CONVERTS A GIVEN TRANSFER FUNCTION TO A SET OF FREQUENCY  0004
CDOMAIN REPRESENTATIONS FOR ANALYSIS AND DESIGN STUDIES. THIS TRANSFER   0005
CFUNCTION IS READ AS A RATIO OF TWO COMPLEX FREQUENCY POLYNOMIALS, WITH  0006
CTWO VARIABLE PARAMETERS ALPHA AND BETA EMBEDDED IN THE COEFFICIENTS.    0007
CTHE PROGRAM THEN TRANSFORMS THIS  TRANSFER FUNCTION INTO THE ANGULAR    0008
CFREQUENCY DOMAIN, YIELDING AN EQUATION IN TERMS OF FOUR VARIABLES:      0009
CALPHA, BETA, MAGNITUDE, AND OMEGA. SEVEN TWO-DIMENSIONAL GRAPHICAL      0010
CREPRESENTATIONS ARE GENERATED BY FIXING ONE VARIABLE AS A CONSTANT;     0011
CASSIGNING DESIRED CURVE VALUES TO ANOTHER; AND LETTING THE REMAINING    0012
CTWO VARIABLES BE THE ORDINATE AND ABCISSA VALUES. A THIRD COEFFICIENT   0013
CVARIABLE PARAMETER, GAMMA, MAY ALSO BE EMBEDDED INTO THE POLYNOMIAL.    0014
CIN THIS CASE, THE ABOVE PROCESS IS REPEATED FOR EACH VALUE OF THE THIRD 0015
CVARIABLE.           .....  DATA CARD INFORMATION FOLLOWS  ......        0016
C*********PROGRAM MODIFICATION**********
C DUE TO ADDITIONAL CORE REQUIREMENTS, PARAM-7 HAS BEEN MODIFIED.
C IN PARAM-7, ONLY THE FOLLOWING COMBINATIONS ARE ALLOWABLE: MAG AND
C PHASE, MAG AND POLAR, MAG AND GAMMA.  CARE MUST BE TAKEN ON LIMITS OF
C XMAGMN AND XMAGMX. (XMAGMN.LT.0.0.LT.XMAGMX) TOTAL MEMORY REQUIREMENT
C IS 240K FOR ALL 7-PARAMS. IF PARAM'S 1-6 ONLY ARE TO BE USED, CARD
C 0143 IN THE DIMENSION STATEMENT MAY BE REPLACED BY A DUMMY CARD:
C ***(S(1,1),T(1,1),U(1,1),V(1,1),W(1,1))*** AND CARD 0276 REPLACED BY
C ***(DO 31 I=1,1)*** AND THE MEMORY REQUIREMENT IS DECREASED TO 100K.
C*****************DATA CARD INFORMATION  SECTION************************ 0017
CCARD FORMAT  SYMBOL        USAGE OF SYMBOL                              0018
C  1   8I10   NRUN    NUMBER OF DESIRED RUNS. EACH MUST HAVE A COMPLETE  0019
C                     SET OF DATA CARDS LISTED BELOW. NOT EVERY SYMBOL   0020
C                     IS USED IN ALL CASES. LEAVE UNUSED SYMBOLS BLANK.  0021
C                     DO NOT OMIT ANY BLANK CARDS, UNLESS SO SPECIFIED.  0022
C                     INSERT ADDITIONAL CARDS BEHIND EACH FULL CARD.     0023
C  2   8I10   KPARAM  NUMBER OF GRAPHICAL PRESENTATIONS TO BE PROGRAMMED 0024
C             JPARAM  NUMERICAL TITLES (1 TO 7) OF EACH PARAM TO BE RUN: 0025
C                     MUST AGREE IN NUMBER WITH KPARAM ABOVE.            0026
C  3   8I10   NORN    ORDER OF NUMERATOR POLYNOMIAL (LIMITED TO 49)      0027
C             NORD    ORDER OF DENOMINATOR POLYNOMIAL (LIMITED TO 49)    0028
C             MODORD  Y-AXIS VARIABLE, AND CURVE VARIABLE CONTROL        0029
C                     MODORD=1: ALPHA IS ORDINATE VARIABLE (PARAM 1-6)   0030
C                     MODORD=2: BETA  IS ORDINATE VARIABLE (PARAM 1-6)   0031
C                     MODORD=1: ALPHA CURVES WILL BE PLOTTED (PARAM 7)   0032
C                     MODORD=2: BETA CURVES WILL BE PLOTTED (PARAM 7)    0033
C             NALPHA  NUMBER OF ALPHA CURVES DESIRED (LIMITED TO 16)     0034
C             NBETA   NUMBER OF BETA CURVES DESIRED (LIMITED TO 16)      0035
C             NMAG    NUMBER OF MAGNITUDE CURVES DESIRED (LIMITED TO 16) 0036
C             NOMEGA  NUMBER OF OMEGA CURVES DESIRED (LIMITED TO 16)     0037
C             NGAMMA  POSITIVE INTEGER DENOTES NUMBER OF THIRD PARAMETER 0038
C                     VALUES TO BE EMBEDDED INTO EACH PARAM RUN. INSERT  0039
C                     COEFFICIENTS FOR THESE GAMMA VALUES ON SEPARATE    0040
C                     CARDS DIRECTLY BEHIND REST OF COEFFICIENTS; SEE#19 0041
```

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C                         NEGATIVE INTEGER DENCTES NUMBER OF PARAM-7 VECTOR      0042
C                         ADDITIONS FOR GENERALIZED BLOCK DIAGRAM STUDIES.        0043
C                         THE (NEGATIVE) VALUE OF NGAMMA DETERMINES HOW MANY      0044
C                         BODE CALCULATIONS WILL BE MADE BEFORE PLOTTING THE      0045
C                         VECTOR RESULTANT AS ONE MAGNITUDE, AND ONE PHASE.       0046
C                         INSERT ADDITIONAL SETS OF COEFFICIENTS AT END .         0047
C  4     8I10   IMAG      MAGNITUDE VALUE FORM CONTROL                            0048
C                         IMAG=0: ALL VALUES IN DECIBELS (PARAM 7 THIS FORM)      0049
C                         IMAG=1 MEANS ALL VALUES IN ACTUAL MAGNITUDE             0050
C                         IMAG=2 MEANS ALL VALUES IN MAGNITUDE SQUARED            0051
C               IWRITE    PRINTOUT CONTROL                                        0052
C                         IWRITE=-1 MEANS NO PRINTOUT                             0053
C                         IWRITE=0 MEANS ONLY INPUT DATA WILL BE PRINTED          0054
C                         IWRITE=1 TO 900 YIELDS INPUT DATA, GRAPHICAL            0055
C                         INFORMATION, AND COMPUTATIONAL RESULTS. THE NUMBER      0056
C                         CF COMPUTATIONS BETWEEN EACH PRINTOUT IS DICTATED       0057
C                         BY THE VALUE  OF IWRITE. IF IWRITE IS GREATER THAN      0058
C                         900, ONLY INPUT DATA & GRAPH INFO. WILL BE PLOTTED      0059
C               IGRAPH    FREQUENCY SCALING CONTROL                               0060
C                         IGRAPH=-2: NO GRAPHS WILL BE PLOTTED.                   0061
C                         IGRAPH=-1:ONLY PARAM 7 WILL BE PLOTTED ON LOG GRID      0062
C                         IGRAPH=0: PARAMS 5&7 WILL BE PLOTTED ON LOG GRID.       0063
C                         IGRAPH=1:PARAMS 4,6,&7 WILL BE PLOTTED ON LOG GRID      0064
C                         IGRAPH=2: PARAMS 4&7 WILL BE PLOTTED ON LOG GRID.       0065
C               IPHASE    PARAM 7 PHASE PLCT CONTROL                              0066
C                         IPHASE=-1 MEANS NO PHASE CURVES                         0067
C                         IPHASE=0: PHASE CURVES SUPERIMPOSED ON MAGNITUDE        0068
C                         IPHASE=1: PHASE CURVES PLOTTED ON SEPARATE GRAPH        0069
C               IQUAD     PARAM 7 PHASE PLOT SCALING CONTROL                      0070
C                         IQUAD=0: PHASE CURVES AUTOMATICALLY SCALED AT 45        0071
C                         DEGREES/INCH, WITH -180 COINCIDENT WITH 0 DB MAG.       0071A
C                         IQUAD=1 TO 100: PHASE CURVES FROM 0 DEGREES TO          0072
C                         -(IQUAD)(LX) DEGREES AT A SCALE OF IQUAD DEG/INCH       0072A
C               IPCLAR    PARAM 7 POLAR PLOT CONTROL                              0073
C                         IPOLAR=0 MEANS NO POLAR PLOT                            0074
C                         IPOLAR=1 GIVES POLAR PLOT ON SEPARATE GRAPH             0075
C               LX        LENGTH OF X-AXIS  (LIMITED TO 9 INCHES)                 0076
C               LY        LENGTH OF Y-AXIS  (LIMITED TO 15 INCHES)                0077
C  5   8F10.3   CONALP    VALUE OF ALPHA WHEN USED AS A CONSTANT                  0078
C               CONBET    VALUE OF BETA WHEN USED AS A CONSTANT                   0079
C               CONMAG    MAGNITUDE VALUE WHEN USED AS A CONSTANT; SEE IMAG       0080
C               CONOME    VALUE OF OMEGA WHEN USED AS A CONSTANT (NOT ZERO)       0081
C**************DATA CARD INFORMATION SECTION (CONTINUED)**************           0082
CCARD FORMAT    SYMBOL          USAGE OF SYMBOL                                   0083
C  6   8F10.3   ALPHMN    MIN. VALUE OF ALPHA WHEN USED AS AN AXIS VARIABLE       0084
C               ALPHMX    MAX. VALUE OF ALPHA WHEN USED AS AN AXIS VARIABLE       0085
C               BETAMN    MIN. VALUE OF BETA WHEN USED AS AN AXIS VARIABLE        0086
C               BETAMX    MAX. VALUE OF BETA WHEN USED AS AN AXIS VARIABLE        0087
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C               XMAGMN   MIN. MAG. VALUE WHEN USED AS AXIS VAR.: SEE IMAG
C               XMAGMX   MAX. MAG. VALUE WHEN USED AS AXIS VAR.: SEE IMAG
C               OMEGMN   MIN. OMEGA VALUE WHEN USED AS AXIS VARIABLE
C               OMEGMX   MAX. OMEGA VALUE WHEN USED AS AXIS VARIABLE
C                        NOTE:IF LOG GRID IS REQUESTED,(SEE IGRAPH), OMEGMN
C                        WILL BE AUTO. CHANGED TO NEAREST POWER OF 10, AND
C                        OMEGMX CHANGED LIKEWISE. THE NUMBER OF DECADES IS
C                        AUTOMATICALLY COMPUTED FROM THESE NEW VALUES.
C   7   8F10.3  ZALPHA   DESIRED VALUES FOR ALPHA CURVES (SEE NALPHA)
C   8   8F10.3  ZBETA    DESIRED VALUES FOR BETA CURVES (SEE NBETA)
C   9   8F10.3  ZMAG     DESIRED VALUES FOR MAGNITUDE CURVES(SEE NMAG&IMAG)
C  10   8F10.3  ZOMEGA   DESIRED POSITIVE OMEGA CURVE VALUES (SEE NOMEGA)
C  11   8F10.3  HA       NUMERATOR CONSTANT COEFFICIENTS IN ASCENDING ORDER
C  12   8F10.3  HAT      NUMERATOR ALPHA COEFFICIENTS IN ASCENDING ORDER
C  13   8F10.3  HBT      NUMERATOR BETA COEFFICIENTS IN ASCENDING ORDER
C  14   8F10.3  HCT      NUM. ALPHA-BETA COEFFICIENTS IN ASCENDING ORDER
C  15   8F10.3  HD       DENOMINATOR CONST. COEFFICIENTS IN ASCENDING ORDER
C  16   8F10.3  HDT      DENOMINATOR ALPHA  COEFFICIENTS IN ASCENDING ORDER
C  17   8F10.3  HET      DENOMINATOR BETA COEFFICIENTS IN ASCENDING ORDER
C  18   8F10.3  HFT      DENOM. ALPHA-BETA COEFFICIENTS IN ASCENDING ORDER
C * * * * * * * * * *   OMIT CARDS # 19 THRU # 27 IF NGAMMA LESS THAN ONE.
C  19   8F10.3  ZGAMMA   VALUES OF THE THIRD PARAMETER; MUST MATCH NGAMMA
C  20   8F10.3  GAMHA    NUM. GAMMA-CONST.  COEFFICIENTS IN ASCENDING ORDER
C  21   8F10.3  GAMHAT   NUM. GAMMA-ALPHA   COEFFICIENTS IN ASCENDING ORDER
C  22   8F10.3  GAMHBT   NUM. GAMMA-BETA    COEFFICIENTS IN ASCENDING ORDER
C  23   8F10.3  GAMHCT   NUM. GAMMA-ALPHA-BETA COEFFS.   IN ASCENDING ORDER
C  24   8F10.3  GAMHD    DEN. GAMMA-CONST. COEFFICIENTS IN ASCENDING ORDER
C  25   8F10.3  GAMHDT   DEN. GAMMA-ALPHA  COEFFICIENTS IN ASCENDING ORDER
C  26   8F10.3  GAMHET   DEN. GAMMA-BETA   COEFFICIENTS IN ASCENDING ORDER
C  27   8F10.3  GAMHFT   DEN. GAMMA-ALPHA-BETA COEFFS.  IN ASCENDING ORDER
C  28   6A8     ITITLE   FIRST LINE GRAPH TITLE (COLUMNS 1THRU48)
C  29   6A8     ITITLE   SECOND LINE OF GRAPH TITLE (COLUMNS 1THRU48)
C                        THE NUMBER OF TITLES FOR PARAM 7 MUST MATCH THE
C                        NUMBER OF GRAPHS REQUESTED.(SEE IPHASE & IPOLAR)
C  30   6A8     JTITLE   1ST 1/2 PHASE TITLE;OMIT UNLESS NPARAM=7&IPHASE=1
C  31   6A8     JTITLE   2ND 1/2 PHASE TITLE;OMIT UNLESS NPARAM=7&IPHASE=1
C  32   6A8     KTITLE   1ST 1/2 POLAR TITLE;OMIT UNLESS NPARAM=7&IPOLAR=1
C  33   6A8     KTITLE   2ND 1/2 POLAR TITLE;OMIT UNLESS NPARAM=7&IPOLAR=1
C*****************************************************************************
C*****************************************************************************
      REAL*8ITITLE(12),JTITLE(12),KTITLE(12),JOKER(9)/1H ,6HPARAM-,7HFUR
     &THER,5HALPHA,4HBETA,5HOMEGA,8HMAGNITUD,5HGAMMA,8HSEPARATE/
      REAL DM/1H1/,LA(66)/4H 0.1,4H 0.2,4H 0.3,4H 0.4,4H 0.5,4H 0.6,4H 0
     &.7,4H 0.8,4H 0.9,4H 1.0,4H 1.0,4H 2.0,4H 3.0,4H 4.0,4H 5.0,4H 6.0,
     &4H 7.0,4H 8.0,4H 9.0,4H10.0,4H10.0,4H20.0,4H30.0,4H40.0,4H50.0,
     & 4H60.0,4H70.0,4H80.0,4H90.0,4H100.,4H100.,4H200.,4H300.,4H400.,
     &4H500.,4H600.,4H700.,4H800.,4H900.,4H1000,4H1000,4H2000,4H3000,
     &4H4000,4H5000,4H6000,4H7000,4H8000,4H9000,1H ,1H1,1H2,1H3,1H4,1H5,
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&1H6,1H7,1H8,1H9,2H10,2H11,2H12,2H13,2H14,2H15,2H16/
 DIMENSION JPARAM(8),A(50),B(50),D(50),E(50),HA(50),HAT(50),HBT(50)
&,HCT(50),HD(50),HDT(50),HET(50),HFT(50),OLDHA(50),OLDHAT(50),OLDHB
&T(50),OLDHCT(50),OLDHD(50),OLDHDT(50),OLDHET(50),OLDHFT(50),GAMHA(
&50),GAMHAT(50),GAMHBT(50),GAMHCT(50),GAMHD(50),GAMHDT(50),GAMHET(5
&0),GAMHFT(50),IC(16),ZALPHA(16),ZBETA(16),ZGAMMA(16),ZMAGSQ(16),ZO
&MEGA(16),X(900),Y(900),
&S(16,900),T(16,900),U(16,900),V(16,900),W(16,900)
 EQUIVALENCE(U,W,S),(V,T)
 CALL CANCEL(2)
1 FORMAT(8I10)
2 FORMAT(10X,8I10)
3 FORMAT(6A8)
4 FORMAT(10X,6A8)
5 FORMAT(8F10.3)
6 FORMAT(10X,1P10E12.3)
C*************************INPUT DATA READ IN **************************
 READ(5,1)NRUN
 DO 1000 IRUN=1,NRUN
 READ(5,1) KPARAM,(JPARAM(I),I=1,KPARAM)
 READ(5,1)NORN,NORD,MODORD,NALPHA,NBETA,NMAG,NOMEGA,NGAMMA
 READ(5,1) IMAG ,IWRITE ,IGRAPH,IPHASE,IQUAD,IPOLAR,LX,LY
 READ(5,5)CONALP,CONBET,CONMAG,CONOME
 READ(5,5)ALPHMN,ALPHMX,BETAMN,BETAMX,XMAGMN,XMAGMX,OMEGMN,OMEGMX
 READ(5,5) (ZALPHA(I),I=1,NALPHA)
 READ(5,5) (ZBETA(I),I=1,NBETA)
 READ(5,5) (ZMAGSQ(I),I=1,NMAG)
 READ(5,5) (ZOMEGA(I),I=1,NOMEGA)
 LOE=NORN+1
 MOE=NORD+1
 READ(5,5) (HA (I), I=1,LOE)
 READ(5,5) (HAT(I), I=1,LOE)
 READ(5,5) (HBT(I), I=1,LOE)
 READ(5,5) (HCT(I), I=1,LOE)
 READ(5,5) (HD (I), I=1,MOE)
 READ(5,5) (HDT(I), I=1,MOE)
 READ(5,5) (HET(I), I=1,MOE)
 READ(5,5) (HFT(I), I=1,MOE)
 IF((NGAMMA).GE.(1)) READ(5,5) (ZGAMMA(I),I=1,NGAMMA)
 IF((NGAMMA).GE.(1)) READ(5,5) (GAMHA (I),I=1,LOE)
 IF((NGAMMA).GE.(1)) READ(5,5) (GAMHAT(I),I=1,LOE)
 IF((NGAMMA).GE.(1)) READ(5,5) (GAMHBT(I),I=1,LOE)
 IF((NGAMMA).GE.(1)) READ(5,5) (GAMHCT(I),I=1,LOE)
 IF((NGAMMA).GE.(1)) READ(5,5) (GAMHD (I),I=1,MOE)
 IF((NGAMMA).GE.(1)) READ(5,5) (GAMHDT(I),I=1,MOE)
 IF((NGAMMA).GE.(1)) READ(5,5) (GAMHET(I),I=1,MOE)
 IF((NGAMMA).GE.(1)) READ(5,5) (GAMHFT(I),I=1,MOE)
 IF((IWRITE).LT.(0)) GO TO 24
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C***************************INPUT DATA PRINT OUT SECTION*****************
      WRITE(6,7) IRUN,NORN,NORD,KPARAM,(JOKER(2),JPARAM(I),I=1,KPARAM)
    7 FORMAT(1H1,10X,'*****  THE INPUT DATA FOR RUN',I2,' IS  *****',//,
     &10X,'CRDER CF NUMERATOR:',I2,//,10X,'ORDER OF DENOMINATOR:',I2,//,
     &10X,'THE FOLLCWING',I2,' GRAPHICAL CCNFIGURATIONS WILL BE COMPUTED
     &:',//,20X,8(A8,I1,3X),/)
      IF((NGAMMA).GE.(1)) WRITE(6,8) NGAMMA,(ZGAMMA(I),I=1,NGAMMA)
    8 FORMAT(10X,I2,' GAMMA, OR THIRD VAR. PARAMETER COEFFICIENTS WILL B
     &E USED.',//,10X,'THE GAMMA VALUES ARE:',1P8E12.3,/,10X,1P8E12.3,/)
      WRITE(6,10)
   10 FORMAT(/,10X,'THE COEFFICIENTS FCR THE NUMERATOR ARE:',/)
      DO 11 I=1,LCE
      J=I-1
      IF(NGAMMA .LE.0) WRITE(6,12) HA(I),HAT(I),HBT(I),HCT(I),J
   12 FORMAT(20X,'(',1PE10.3,'+',1PE10.3,'*ALPHA + ' ,1PE10.3,'*BETA + '
     &,1PE10.3,'*ALPHA*BETA ) * S**(',I2,')',/)
      IF(NGAMMA .GT.0) WRITE(6, 9) HA(I),GAMHA(I),HAT(I),GAMHAT(I),HBT(I
     &),GAMHBT(I),HCT(I),GAMHCT(I),J
    9 FORMAT(15X,'((',1PE10.3,'+',1PE10.3,'*GAMMA)+(',1PE10.3,'+',1PE10.
     &3,'*GAMMA)*ALPHA ....',//,20X,'+(',1PE10.3,'+',1PE10.3,'*GAMMA)*BE
     &TA+(',1PE10.3,'+',1PE10.3,'*GAMMA)*ALPHA*BETA)**(',I2,')',/)
   11 CONTINUE
      WRITE(6,13)
   13 FORMAT(/10X,'THE COEFFICIENTS FCR THE DENOMINATCR ARE:',/)
      DO 14 I=1,MOE
      J=I-1
      IF(NGAMMA .LE.0) WRITE(6,12) HD(I),HCT(I),HET(I),HFT(I),J
      IF(NGAMMA .GT.0) WRITE(6, 9) HD(I),GAMHD(I),HDT(I),GAMHDT(I),HET(I
     &),GAMHET(I),HFT(I),GAMHFT(I),J
   14 CONTINUE
      WRITE(6,15)
   15 FORMAT(15X,'  NORN       NORC       MODORD     NALPHA      NBETA     NMA
     &G      NOMEGA   NGAMMA ',/)
      WRITE(6,2 ) NORN,NORD,MODORD,NALPHA,NBETA,NMAG,NOMEGA,NGAMMA
      WRITE(6,16)
   16 FORMAT(/,17X,'IMAG       IWRITE    IGRAPH     IPHASE      IQUAD     IPO
     &LAR      LXAXIS     LYAXIS  ',/)
      WRITE(6,2)  IMAG ,IWRITE,IGRAPH,IPHASE,IQUAD,IPOLAR,LX,LY
      WRITE(6,17)
   17 FORMAT(/,12X,' CCNALPHA     CCNBETA       CONMAG       CONOMEGA   ',/)
      WRITE(6,6 )CONALP,CONBET,CONMAG,CONCME
      WRITE(6,18)
   18 FORMAT(/,12X,' ALPHMIN      ALPHMAX     BETAMIN       BETAMAX      XMAG
     &MIN      XMAGMAX       OMEGAMIN       CMEGAMAX  ',/)
      WRITE(6,6)ALPHMN,ALPHMX,BETAMN,BETAMX,XMAGMN,XMAGMX,OMEGMN,OMEGMX
      WRITE(6,19) JOKER(4),(ZALPHA(I),I=1,NALPHA)
      WRITE(6,19) JCKER(5),(ZBETA (I),I=1,NBETA )
      WRITE(6,19) JOKER(7),(ZMAGSQ(I),I=1,NMAG)
```

```
      WRITE(6,19) JCKER(6),(ZOMEGA(I),I=1,NCMEGA)
   19 FORMAT(/,10X,A8,'CURVE VALUES:',1P8E12.3,/31X,1P8E12.3)
      IF(IWRITE) 20,20,22
   20 WRITE(6,21)
   21 FORMAT(/,10X,'THERE WILL BE NO PRINTCUT CF COMPUTED RESULTS')
      GO TC 24
   22 WRITE(6,23)IWRITE
   23 FORMAT(/,10X,'COMPUTED RESULTS WILL BE PRINTED EVERY',I3,' STEPS')
C*****************INITIALIZING COMPUTATIONAL SETTINGS*****************
      NPAR7=IABS(NGAMMA)
   24 IF(MCDORD.GE.2) GO TO  25
      DUMMY1=BETAMN
      DUMMY2=BETAMX
      BETAMN=ALPHMN
      BETAMX=ALPHMX
      ALPHMN=DUMMY1
      ALPHMX=DUMMY2
   25 DO 1000 IPARAM=1,KPARAM
      NPARAM=JPARAM(IPARAM)
      MGAMMA=NGAMMA+1
      DO 1000 IGAMMA=1,MGAMMA
      GAMMA=ZGAMMA(IGAMMA)
      IF(NGAMMA.GE.1) GO TO 517
   26 XS=(ALPHMX-ALPHMN)/LX
      IF((ABS(XS)).LE.(1.0E-10)) XS=1.CE-10
      YS=(BETAMX-BETAMN)/LY
      IF((ABS(YS)).LE.(1.0E-10)) YS=1.CE-10
      IX=-BETAMN/YS
      IY=-ALPHMN/XS
      MOD=1
      JCURVE=0
      IPAR7=1
      READ(5,3)(ITITLE(I),I=1,12)
      IF(NPARAM.NE.7)GO TO 34
      IF(IMAG-1)27,28,29
   27 DBMAX=XMAGMX
      DBMIN=XMAGMN
      GO TO 30
   28 DBMAX=20*ALCG10(XMAGMX)
      DBMIN=20*ALCG10(XMAGMN)
      GO TC 30
   29 DBMAX=10*ALCG10(XMAGMX)
      DBMIN=10*ALCG10(XMAGMN)
   30 XS=(DBMAX-DBMIN)/LX
      XX=EXP(XS/(20.0*.4342945))
      IY=DBMAX/XS
      DO 31 I=1,16
      S(I,1)=0.0
```

```
      T(I,1)=0.0
      U(I,1)=0.0
      V(I,1)=0.0
   31 W(I,1)=0.0
      IF(IPHASE.LE.0) GO TO 32
      READ(5,3)(JTITLE(I),I=1,12)
   32 IF((IPOLAR).LE.(0)) GO TO 33
      READ(5,3)(KTITLE(I),I=1,12)
   33 IF(MODORD-2)45,44,44
   34 GO TO (35,40,41,46,47,51,59,59),NPARAM
   35 NLIMIT=NOMEGA
   36 IF(IMAG-1)37,38,39
   37 DUMMY1=10.0**(CONMAG/10.0)
      IF(NPARAM-1)59,59,43
   38 DUMMY1=CONMAG*CONMAG
      IF(NPARAM-1)59,59,43
   39 DUMMY1=CONMAG
      IF(NPARAM-1)59,59,43
   40 NLIMIT=NMAG
   41 OMEGA=CONOME
      IF(NPARAM-2)59,59,42
   42 XS=(XMAGMX-XMAGMN)/LX
      IY=-XMAGMN/XS
   43 IF(MODORD-2)44,45,45
   44 NLIMIT=NBETA
      IF(NPARAM-6)59,53,47
   45 NLIMIT=NALPHA
      IF(NPARAM-6)59,53,47
   46 NLIMIT=NMAG
      XS=(OMEGMX-OMEGMN)/LX
      IY=-OMEGMN/XS
   47 IF(MODORD-2)48,49,49
   48 CONST=CONBET
      IF(NPARAM-4)52,52,50
   49 CONST=CONALP
      IF(NPARAM-4)52,52,50
   50 IF(NPARAM.GE.7) GO TO 55
      XS=(XMAGMX-XMAGMN)/LX
      IY=-XMAGMN/XS
      NLIMIT=NOMEGA
      GO TO 59
   51 XS=(OMEGMX-OMEGMN)/LX
      IY=-OMEGMN/XS
      GO TO 36
   52 IF(IGRAPH)59,59,54
   53 IF((IGRAPH.LT.0).OR.(IGRAPH.GT.1)) GO TO 59
   54 IY=LX-IX
   55 IF(IGRAPH.LE.-2) GO TO 59
```

```
C*******************LOG GRID GRAPHICAL CONSTRUCTION*********************
      IF(OMEGMN.LT.1000.)YLOGMN=100.
      IF(OMEGMN.LT.100. )YLOGMN=10.
      IF(OMEGMN.LT.10.  )YLOGMN=1.
      IF(OMEGMN.LT.1.   )YLOGMN=0.1
      IF(OMEGMN.LT.0.1   )YLOGMN=0.01
      IF(OMEGMN.LT.0.01  )YLOGMN=0.001
      IF(OMEGMX.GT.0.01  )YLOGMX=0.1
      IF(OMEGMX.GT.0.1    )YLOGMX=1.0
      IF(OMEGMX.GT.1.0 )YLOGMX=10.0
      IF(OMEGMX.GT.10.0)YLOGMX=100.0
      IF(OMEGMX.GT.100.)YLOGMX=1000.0
      IF(OMEGMX.GT.1000.)YLOGMX=10000.0
      DUMMY3= YLOGMX/YLOGMN
      NUMDEC=ALOG10(DUMMY3)
      IX=0
      DUMMY4=NUMDEC
      DUMMY5=LY
      YS=DUMMY4/DUMMY5
      IFINAL=LX+1
      JFINAL=LY+1
      DO 57  IA=1,IFINAL
      DO 56  JA=1,JFINAL
      X(JA)=XS*(IA-1-IY)
   56 Y(JA)=YS*(JA-1)
      CALL DRAW(JA,X,Y,MOD,0,LA(50),ITITLE,XS,YS ,0,IY,2,2,LX,LY,0,LAST)
   57 MOD=2
      DO 58 IB=1,NUMDEC
      DO 58 JB=1,10
      J=ALOG10(YLOGMN)*10.+10.+(IB-1)*10.+JB
      YLOG=JB
      DO 1058 KB=1,IFINAL
      X(KB)=XS*(KB-1-IY)
 1058 Y(KB)=ALOG10(YLOG)+IB-1
   58 CALL DRAW(KB,X,Y,MOD,0,LA(J),ITITLE,XS,YS ,0,IY,2,2,LX,LY,0,LAST)
      IF((IPAR7).NE.(1)) GO TO 532
   59 IF(IWRITE.LE.0) GO TO 200
C*****************GRAPHICAL  INFORMATION PRINTOUT SECTION**************
      WRITE(6,60) IRUN, NPARAM
   60 FORMAT(1H1,10X,'**********THE PRELIMINARY DATA FOR RUN NUMBER',I2,
     &', PARAMETER NUMBER',I2,' IS    **********',///)
      IF(NGAMMA.GE.1) WRITE (6,61) ZGAMMA(IGAMMA)
   61 FORMAT(10X,'THE FOLLOWING CALCULATIONS WILL BE BASED ON THE THIRD
     &VARIABLE PARAMETER VALUE , GAMMA=',1PE12.5,//)
      WRITE (6,62) (ITITLE(I),I=1,12)
   62 FORMAT(10X,'THE GRAPH WILL BE TITLED:',//,10X,6A8,//,10X,6A8,/)
      IF(NPARAM.NE.7) GO TO 80
      WRITE(6,83)
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      WRITE(6,  63)DBMIN,DBMAX,XS
   63 FORMAT(10X,'MAGNITUDE, THE DEPENDENT VARIABLE, WILL BE PLOTTED ON
     &THE ORDINATE FROM',F10.4,' TO',F10.4,' AT',F10.4,' DB/DIVISION',/)
      IF(IPHASE) 73,64,66
   64 WRITE(6,65)
   65 FORMAT(10X,'PHASE CURVES WILL BE PLOTTED ON MAGNITUDE GRAPH',/)
      GO TO 68
   66 WRITE(6,67)
   67 FORMAT(10X,'PHASE CURVES WILL BE PLOTTED ON SEPARATE GRAPH',/)
   68 IF(IQUAD.GT.0) GO TO 70
      WRITE(6,69)
   69 FORMAT(10X,'AUTOMATICALLY SCALED AT 45DEG/IN,-180 ON ODB MAG.',/)
      GO TO 73
   70 WRITE(6,71) IQUAD
   71 FORMAT(10X,'WITH A SCALE OF',I3,' DEG/IN, STARTING AT 0 DEG.',/)
   72 WRITE(6,62) (JTITLE(I),I=1,12)
   73 IF(IPOLAR.LE.0) GO TO 75
      WRITE(6,74) XX
   74 FORMAT(10X,'A POLAR PLOT OF MAGNITUDE VS. PHASE WILL BE PLOTTED, W
     &ITH MAGNITUDE SCALED AT',F10.5,' UNITS PER INCH',/)
      WRITE(6,62) (KTITLE(I),I=1,12)
   75 IF(MODORD-2)127,121,121
   76 FORMAT(10X,A8,', THE   DEPENDENT VARIABLE, WILL BE PLOTTED ON THE
     &ORDINATE FROM',1PE10.3,' TO',1PE10.3,' WITH A SCALE OF',1PE10.3,/)
   77 FORMAT(10X,A8,', THE INDEPENDENT VARIABLE, WILL BE PLOTTED ON THE
     &ABCISSA, FROM',1PE10.3,' TO',1PE10.3,' WITH A SCALE OF',1PE10.3,/)
   78 FORMAT(10X,A8,' WILL BE HELD CONSTANT AT THE VALUE OF',1PE12.5,/)
   79 FORMAT(10X,A8,' WILL BE GRAPHED AS ',I3,' CURVES, CONSISTING OF TH
     &E FOLLOWING VALUES:',/,16(20X,'CURVE',I3,' = ',1PE12.5,/))
   80 IF(IMAG.EQ.0) WRITE(6,83)
      IF(IMAG.EQ.1) WRITE(6,84)
      IF(IMAG.EQ.2) WRITE(6,85)
   81 IF(MODORD-2) 82,  94,1000
   82 WRITE(6,76) JOKER(4),BETAMN,BETAMX, YS
   83 FORMAT(10X,'ALL MAGNITUDE COMPUTATIONS WILL BE IN DECIBELS',/)
   84 FORMAT(10X,'ALL MAGNITUDE COMPUTATIONS ARE ACTUAL MAGNITUDE',/)
   85 FORMAT(10X,'ALL MAGNITUDE COMPUTATIONS ARE MAGNITUDE SQUARED',/)
      GO TO (86,86,121,130,130,121,200,200),NPARAM
   86 WRITE(6,77) JOKER(5),ALPHMN,ALPHMX,XS
      IF(NPARAM-1) 88,88,99
   88 WRITE(6,78) JOKER(7),CONMAG
   90 WRITE(6,79)JOKER(6),NOMEGA,(I,ZOMEGA(I),I=1,NOMEGA)
      IF(NPARAM-5)200,125,200
   94 WRITE(6,76) JOKER(5),BETAMN,BETAMX, YS
   96 GO TO(97,97,127,111,111,127,200,200),NPARAM
   97 WRITE(6,77) JOKER(4),ALPHMN,ALPHMX,XS
      IF(NPARAM-1)88,88,99
   99 WRITE(6,78) JOKER(6),CONOME
```

```
101  WRITE(6,79)JOKER(7),NMAG,(I,ZMAGSQ(I),I=1,NMAG)
     IF(NPARAM-4)200,139,200
111  WRITE(6,78) JOKER(4),CONALP
113  WRITE(6,114)
114  FORMAT(10X,'THE COMBINED COEFFICIENTS OF THE NUMERATOR ARE',/)
115  DO 116 I=1,LOE
     A(I)=HA(I)+HAT(I)*CONST
     B(I)=HBT(I)+HCT(I)*CONST
     KA=I-1
116  WRITE(6,117) A(I),B(I),KA
117  FORMAT(20X,1H(,E10.4,3H + , E10.4,16H * BETA ) * S**(,I2,1H)/)
     WRITE(6,136)
     DO 119 I=1,MOE
     D(I)=HD(I)+HDT(I)*CONST
     E(I)=HET(I)+HFT(I)*CONST
     KA=I-1
119  WRITE(6,117) D(I),E(I),KA
120  GO TO (200,200,200,101, 90,200,139,200),NPARAM
121  WRITE(6,79)JOKER(5),NBETA,(I,ZBETA(I),I=1,NBETA)
     IF(NPARAM-6)124,138,111
124  WRITE(6,78) JOKER(6),CONOME
125  WRITE(6,77) JOKER(7),XMAGMN,XMAGMX,XS
     GO TO 200
127  WRITE(6,79)JOKER(4),NALPHA,(I,ZALPHA(I),I=1,NALPHA)
     IF(NPARAM-6)124,138,130
130  WRITE(6,78) JOKER(5),CONBET
131  FORMAT(10X,'BETA  WILL BE HELD CONSTANT AT THE  VALUE OF',F10.4,/)
132  WRITE(6,114)
     DO 134 I=1,LOE
     A(I)=HA(I)+HBT(I)*CONST
     B(I)=HAT(I)+HCT(I)*CONST
     KA=I-1
134  WRITE(6,135)A(I),B(I),KA
135  FORMAT(20X,1H(,E10.4,3H + ,E10.4,17H * ALPHA ) * S**(,I2,1H)/)
     WRITE(6,136)
136  FORMAT(10X,'THE COMBINED COEFFICIENTS OF THE DENOMINATOR ARE-',/)
     DO 137 I=1,MOE
     D(I)=HD(I)+HET(I)*CONST
     E(I)=HDT(I)+HFT(I)*CONST
     KA=I-1
137  WRITE(6,135) D(I),E(I),KA
     GO TO 120
138  WRITE(6,78) JOKER(7),CONMAG
139  WRITE(6,140)
140  FORMAT(10X,'OMEGA,THE INDEPENDENT VARIABLE,WILL BE PLOTTED ON THE
    &ABCISSA',/)
     IF(NPARAM-6)141,143,144
141  IF(IGRAPH)142,142,144
```

```
142 WRITE(6,146) CMEGMN,OMEGMX,XS
    GO TO 200
143 IF((IGRAPH.LT.0).OR.(IGRAPH.GT.1)) GO TO 142
144 WRITE(6,145)NUMDEC,YLOGMN
145 FORMAT(10X,'WITH A SPAN OF',I2,' DECADES,STARTING AT',F10.4,//,30X
   &,'*****NOTE*****',//,10X,'FOR THIS LOG PLOT, IT WILL BE NECESSARY
   &TO SHIFT THE ORIGIN TO THE LOWER RIGHT HAND CORNER,',//,10X,'WITH
   &THE ABCISSA RUNNING UP THE RIGHT HAND EDGE, AND THE ORDINATE INCRE
   &ASING TOWARDS THE LEFT ALONG THE BOTTOM'//)
146 FORMAT(10X,'FROM THE INITIAL VALUE OF',1PE10.3,'TO THE FINAL VALUE
   & OF',1PE10.3,' WITH A SCALE OF ',1PE10.3,' RADIANS PER INCH',/)
C********************MAIN COMPUTATIONAL SECTION DO LOOPS****************
200 IF((IWRITE.LT.1).OR.(IWRITE.GT.900)) GO TO 203
    WRITE(6, 201)IRUN,NPARAM,IWRITE
201 FORMAT(1H1,10X,'*********THE OUTPUT DATA FOR RUN NUMBER',I2,', PAR
   &AMETER NUMBER',I2,' IS**********',////,10X,I2,' COMPUTATIONS WILL
   &BE PERFORMED BETWEEN EACH PRINTOUT',///)
    IF(NGAMMA.GT.0) WRITE(6,202) ZGAMMA(IGAMMA)
202 FORMAT(10X,'THE VALUE OF GAMMA, THE THIRD VARIABLE IS:',1PE10.3,/)
203 DO 520 LOM=1,NLIMIT
    JCURVE=0
    DO 519 ICOEF=1,3,2
    ZSIGN=ICOEF-2
    IF((IWRITE.LT.1).OR.(IWRITE.GT.900)) GO TO 208
    WRITE(6,205 )LOM
205 FORMAT(//,10X,'COMPUTATIONAL RESULTS FOR CURVE NUMBER',I3)
    IF (NPARAM-7)206,208,208
206 WRITE(6,207 )ZSIGN
207 FORMAT(1H+,55X,' USING A QUADRATIC FORMULA COEFFICIENT OF',F4.1,/)
208 IP=0
    YMAGPK=0.0
    OMEGPK=0.0
    DO 518 INCRMT=1,900
    GO TO (209,216,253,216,212,254,257,1000),NPARAM
209 OMEGA=ZOMEGA(LOM)
211 CONST=ALPHMN+(INCRMT-1)*(ALPHMX-ALPHMN)/899.0
212 IF(MODORD-2)260,213,213
213 DO 214  I=1,LCE
    A(I)=HA(I)+HAT(I)*CONST
214 B(I)=HBT(I)+HCT(I)*CONST
    DO 215 I =1,MCE
    D(I)=HD(I)+HDT(I)*CONST
215 E(I)=HET(I)+HFT(I)*CONST
    GO TO 263
216 IF(IMAG-1)217,218,219
217 DUMMY1=10.0**(ZMAGSQ(LOM)/10.0)
    GO TO 220
218 DUMMY1=ZMAGSQ(LOM)*ZMAGSQ(LOM)
```

```
      GO TC 220
  219 DUMMY1=ZMAGSQ(LOM)
  220 IF(NPARAM-2)211,211,212
  253 OMEGA=CONOME
  254 IF(MODORD-2)255,256,1000
  255 CONST=ZBETA(LOM)
      GO TC 260
  256 CONST=ZALPHA(LOM)
      GO TO 213
  257 IF(MODORD-2)259,258,258
  258 BETA=ZBETA(LOM)
      CONST=CCNALP
      GO TO 213
  259 BETA=ZALPHA(LCM)
      CONST=CCNBET
  260 DO 261  I=1,LCE
      A(I)=HA(I)+HBT(I)*CONST
  261 B(I)=HAT(I)+HCT(I)*CONST
      DO 262  I=1,MOE
      D(I)=HD(I)+HET(I)*CONST
  262 E(I)=HDT(I)+HFT(I)*CONST
  263 GO TO (350,350,275, 290,264 ,291,294,1000),NPARAM
  264 OMEGA=ZOMEGA(LOM)
  275 ZMAG=XMAGMN+(INCRMT-1)*(XMAGMX-XMAGMN)/899.0
      IF(IMAG-1)276,277,278
  276 DUMMY1=10.0**(ZMAG/10.0)
      GO TO 350
  277 DUMMY1=ZMAG*ZMAG
      GO TO 350
  278 DUMMY1=ZMAG
      GO TO 350
  290 IF(IGRAPH)292,292,294
  291 IF((IGRAPH.GE.0).AND.(IGRAPH.LE.1)) GO TO 294
  292 OMEGA=OMEGMN+INCRMT*(OMEGMX-OMEGMN)/900.0
      GO TO 350
  294 WINC=1.0+(INCRMT-1)*NUMDEC/899.0
      OMEGA=(YLOGMN*10.0**WINC)/10.0
C*************NUMERATCR COEFFICIENT CALCULATIONS*********************
  350 SNB2=0.0
      SNB1=0.0
      SNB0=0.0
      SUMN=0.0
      NDOUN=2*NORN
      DO 373 ILOOP=1,4
      DO 373 NP=2,LOE
      DO 373 MQ=1,NORN
      MP=MQ-1
      NQ=NP-1
```

```
      IF(((NP+MQ).GT.NDOUN).AND.(NORN.NE.1)) GO TO 373
  361 IF((MP+NQ)-NORN)362,362,373
  362 IF(NP-MP)373,373,363
  363 IF(MP)364,364,366
  364 C=1.0
      GO TO 367
  366 C=2.0
  367 GO TO (368,369,370,371),ILOOP
  368 CSNB2 = (C*B(NP+MP)*B(NP-MP))*OMEGA**(2*NQ)*(-1.0)**(MQ+1)
      SNB2 = SNB2 + CSNB2
      GO TO 373
  369 CSNB1 = (C*A(NP+MP)*B(NP-MP)+C*A(NP-MP)*B(NP+MP))*OMEGA**(2*NQ)*(-
     11.0)**(MQ+1)
      SNB1 = SNB1 + CSNB1
      GO TO 373
  370 CSNBO = (C*A(NP+MP)*A(NP-MP))*CMEGA**(2*NQ)*(-1.0)**(MQ+1)
      SNBO = SNBC + CSNBO
      GO TC 373
  371 COEFN = (C*A(NP+MP)*A(NP-MP)+C*A(NP+MP)*B(NP-MP)*BETA+C*A(NP-MP)*B
     1(NP+MP)*BETA+C*B(NP+MP)*B(NP-MP)*BETA*BETA)*OMEGA**(2*NQ)*(-1.0)**
     2(MQ+1)
      SUMN = SUMN + COEFN
  373 CONTINUE
      SNB2 = SNB2 + B(1)*B(1)
      SNB1 = SNB1 + 2.0*A(1)*B(1)
      SNBO=SNBC+A(1)*A(1)
      SUMN = SUMN + A(1)*A(1)+2.0*A(1)*B(1)*BETA+B(1)*B(1)*BETA*BETA
C************DENOMINATOR COEFFICIENT CALCULATIONS*********************
      SDB2=0.0
      SDB1=0.
      SDBO=0.
      SUMD=C.
      NDOUD=2*NORD
      DO 385 ILCCP=1,4
      DO 385 NP=2,MOE
      DO 385 MQ=1,NORD
      MP=MQ-1
      NQ=NP-1
      IF(((NP+MQ).GT.NDOUD).AND.(NORD.NE.1))  GO TO 385
  375 IF((MP+NQ)-NORD)376,376,385
  376 IF(NP-MP)385,385,377
  377 IF(MP)378,378,379
  378 C=1.C
      GO TC 380
  379 C=2.0
  380 GO TO (381,382,383,384),ILOOP
  381 CSDB2 =  C*E(NP+MP)*E(NP-MP)*OMEGA**(2*NQ)*(-1.0)**(MQ+1)
      SDB2 = SDB2 + CSDB2
```

```
      GO TC 385
  382 CSDB1 = (C*D(NP+MP)*E(NP-MP)+C*D(NP-MP)*E(NP+MP))*OMEGA**(2*NQ)*(-
     11.0)**(MQ+1)
      SDB1 = SDB1 + CSDB1
      GO TC 385
  383 CSDB0 = (C*D(NP+MP)*D(NP-MP))*OMEGA**(2*NQ)*(-1.0)**(MQ+1)
      SDB0 = SDB0 + CSDB0
      GO TO 385
  384 COEFD = (C*D(NP+MP)*D(NP-MP)+C*D(NP+MP)*E(NP-MP)*BETA+C*D(NP-MP)*E
     1(NP+MP)*BETA+C*E(NP+MP)*E(NP-MP)*BETA*BETA)*OMEGA**(2*NQ)*(-1.0)**
     2(MQ+1)
      SUMD = SUMD + CCEFD
  385 CONTINUE
      SUMD = SUMD + D(1)*D(1)+2.0*D(1)*E(1)*BETA+E(1)*E(1)*BETA*BETA
      SDB2 = SDB2 + E(1)*E(1)
      SDB1 = SCB1 + 2.0*D(1)*E(1)
      SDB0 = SDB0 + D(1)*D(1)
      IF(NPARAM-7)413, 386,1000
C**********************PARAM 7 PHASE CCMPUTATIONAL SECTION***********
  386 TOE=LOE/2.0
      PNEVEN=0.
      PDEVEN=0.
      PNODD=0.
      PDODD=0.
      IF(NCRN.EQ.0) GO TO 393
      JOE=LCE/2
      IF(JCE-TCE)387,388,388
  387 JON=(LOE+1)/2
      MON=JCN
      GO TO 389
  388 JCN=(LOE+2)/2
      MON=JCN-1
      IF(NCRN-1)393,389,389
  389 DO 390 I=2,JON
  390 PNODD=PNODD+(A(2*I-2)+B(2*I-2)*BETA)*OMEGA**(2*I-3)*(-1.0)**(I)
      IF(NORN-1)393,393,391
  391 DO 392 I=2,MON
  392 PNEVEN=PNEVEN+(A(2*I-1)+B(2*I-1)*BETA)*OMEGA**(2*I-2)*(-1.)**(I-1)
  393 PNEVEN=PNEVEN+A(1)+B(1)*BETA
      TOE=MCE/2.0
      JOE=MCE/2
      IF(JOE-TOE)394,395,395
  394 JON=(MOE+1)/2
      MON=JCN
      GO TO 396
  395 JON=(MOE+2)/2
      MON=JCN-1
      IF(NCRD-1)400,396,396
```

```
  396 DO 397 I=2,JCN                                                        0667
  397 PDODD=PDODD+(C(2*I-2)+E(2*I-2)*BETA)*OMEGA**(2*I-3)*(-1.0)**(I)      0668
      IF(NORD-1)400,400,398                                                 0669
  398 DO 399 I=2,MON                                                        067C
  399 PDEVEN=PDEVEN+(D(2*I-1)+E(2*I-1)*BETA)*OMEGA**(2*I-2)*(-1.)**(I-1)    0671
  400 PDEVEN=PDEVEN+D(1)+E(1)*BETA                                          0672
      TANGIM=PDEVEN*PNODD-PNEVEN*PDODD                                      C673
      TANGRE=PNEVEN*PDEVEN-PNODD*PDODD                                      0674
      TANG=TANGIM/TANGRE                                                    0675
      RAD=ATAN(TANG)                                                        0676
      IF((TANGRE.LT.0.0).AND.(TANGIM.LT.0.0)) JQUAD=1                       0677
      IF((TANGRE.LT.0.0).AND.(TANGIM.GT.0.0)) JQUAD=-1                      0677A
      IF(TANGRE.LT.0.0) RAD=JQUAD*(3.141593+JQUAD*RAD)                      0677B
      IF((TANGRE.GT.0.0).AND.(TANGIM.LT.0.0)) JQUAD=-1
      IF((TANGRE.GT.0.0).AND.(TANGIM.GT.0.0)) JQUAD=1
  402 YMAGSQ=ABS(SUMN/SUMD)                                                 0678
      YMAG=SQRT(YMAGSQ)                                                     0679
      YMAGDB=20.0*.4342945*ALOG(YMAG)                                       0680
      YMAGAB=ABS(YMAGDB)                                                    0681
      IF(YMAGPK-YMAGDB)403,404,404                                          0682
  403 YMAGPK=YMAGDB                                                         0683
      OMEGPK=OMEGA                                                          0684
  404 IF(DBMAX-YMAGDB)405,406,406                                           0685
  405 YMAGDB=DBMAX                                                          0686
  406 IF(YMAGDB-DBMIN)407,408,408                                           0687
  407 YMAGDB=DBMIN                                                          0688
  408 IF((XS*LX )-YMAGAB)409,410,410                                        0689
  409 YMAGAB=LX*XS                                                          0690
  410 GO TO 426                                                             0691
C****************QUADRATIC FORMULA SOLN. OF DEPENDENT VARIABLE, PARAM1-6   0692
  413 ACO=DUMMY1*SDB2-SNB2                                                  0693
  416 BCO=DUMMY1*SDB1-SNB1                                                  0694
      CCO=DUMMY1*SDB0-SNB0                                                  0695
      IF(ABS(ACO).LT.1.0E-35) GO TO 425                                     0696
      DPP=BCO*BCO-4.0*ACO*CCO                                               0697
      IF(DPP.LT.0.0) GO TO 425                                              0698
      DSQR=ZSIGN*SQRT(DPP)                                                  0699
      BETA=(DSQR-BCO)/(2.0*ACO)                                             0700
      IF(BETA-BETAMN)425,426,424                                            0701
  424 IF(BETA-BETAMX)426,426,425                                            0702
  425 IF(IP-1)500,505,507                                                   0703
  426 IF((IWRITE.LT.1).OR.(IWRITE.GT.900)) GO TO 450                        0704
C******************COMPUTED DATA PRINTOUT SECTION*********************      0705
      ISTEP=ISTEP+1                                                         0706
      IF(IP)428,428,427                                                     0707
  427 IF(ISTEP-IWRITE)450,430,430                                           0708
  428 WRITE(6,429)LCM                                                       0709
  429 FORMAT(/,10X,'    ALPHA        BETA      MAGNITUDE        OMEGA       0710
```

```
     & TANGENT   :  CURVE NUMBER',I2,/)
 430 IF(MODORD.EQ.2) GO TO 431
     GO TO (433,435,436,437,439,441,443,1000),NPARAM
 431 GO TO (432,434,445,438,440,442,444,1000),NPARAM
 432 WRITE(6,6)    CONST,BETA,CONMAG,OMEGA
     GO TO 448
 433 WRITE(6,6)    BETA,CONST,CONMAG,OMEGA
     GO TO 448
 434 WRITE(6,6)    CONST,BETA,ZMAGSQ(LCM),OMEGA
     GO TO 448
 435 WRITE(6,6)    BETA,CONST,ZMAGSQ(LCM),OMEGA
     GO TO 448
 436 WRITE(6,6)    BETA,CONST,ZMAG,OMEGA
     GO TO 448
 437 WRITE(6,6)    BETA,CONBET,ZMAGSQ(LCM),OMEGA
     GO TO 448
 438 WRITE(6,6)    CONALP,BETA,ZMAGSQ(LOM),OMEGA
     GO TO 448
 439 WRITE(6,6)    BETA,CONBET,ZMAG,OMEGA
     GO TO 448
 440 WRITE(6,6)    CONALP,BETA,ZMAG,OMEGA
     GO TO 448
 441 WRITE(6,6)    BETA,CONST,CONMAG,OMEGA
     GO TO 448
 442 WRITE(6,6)    CONST,BETA,CONMAG,OMEGA
     GO TO 448
 443 WRITE(6,6)   ZALPHA(LCM),CONBET,YMAGDB,OMEGA,TANG
     GO TO 448
 444 WRITE(6,6)   CONALP,ZBETA(LOM),YMAGDB,OMEGA,TANG
     GO TO 448
 445 WRITE(6,6)    CONST,BETA,ZMAG,OMEGA
 448 ISTEP=0
C******************GRAPHICAL ARRAY STORAGE SECTION*******************
 450 IP=IP+1
     Y(IP)= BETA
 452 GO TO (453,453,460,470,460,470,475 ,1000),NPARAM
 453 X(IP)= CONST
     GO TO 500
 460 X(IP)=ZMAG
     GO TO 500
 470 X(IP)=OMEGA
     IF(NPARAM-6)471,472,475
 471 IF(IGRAPH)500,500,474
 472 IF((IGRAPH.LT.0).OR.(IGRAPH.GT.1)) GO TO 500
 474 X(IP)=LX-BETA
     GO TO 479
 475 X(IP)=-YMAGDB
     IC(LOM)=IP
```

```
      IF((IPHASE.GE.0).OR.(NGAMMA.LT.0)) GO TO 4475                     0759
      S(LOM,IP)=(ABS(YMAG)*COS(RAD))/XX
      T(LOM,IP)=(ABS(YMAG)*SIN(RAD))/XX
      GO TO 479                                                         0762
 4475 V(LOM,IP)=(ALOG10(OMEGA/YLOGMN)*LY/NUMDEC)*YS                     0763
      IF(NGAMMA.LT.0) GO TO 4477                                        0764
      IF(IQUAD.LE.0) GO TO 476                                          0765
      IF(RAD.LE.0.0) W(LOM,IP)=JQUAD*(IY-JQUAD*RAD*57.29256/IQUAD)*XS   0766
      IF(RAD.GT.0.0) IZ=8-IY                                            0766A
      IF(RAD.GT.0.0) W(LOM,IP)=JQUAD*(IZ-JQUAD*RAD*57.29256/IQUAD)*XS   0766B
      GO TO 477                                                         0767
  476 W(LOM,IP)=JQUAD*((3.141593-JQUAD*RAD)/0.7853981)*XS               0766
  477 IF(W(LOM,IP).GT.(-DBMIN)) W(LOM,IP)=-DBMIN                        0769
      IF(W(LOM,IP).LT.(-DBMAX)) W(LOM,IP)=-DBMAX                        0770
      GO TO 479                                                         0771
 4477 U(LOM,IP)=U(LOM,IP)+YMAG                                          0772
  479 Y(IP)=(ALOG10(OMEGA/YLOGMN)*LY/NUMDEC)*YS                         0774
  500 IF(INCRMT.LT.900) GO TO 518                                       0775
C************************GRAPHICAL CURVE PLOTTING********************   0776
      IF(IP-1)501,505,507                                               0777
  501 IF(JCURVE.GT.0) GO TO 503                                         0778
      WRITE(6,502) LOM                                                  0779
  502 FORMAT(10X,'NO GRAPH POINTS WERE FOUND FOR CURVE NUMBER ',I2)     0780
      IF(NPARAM.NE.7) WRITE(6,207) ZSIGN                                0780A
      GO TO 518                                                         0781
  503 WRITE(6, 504)LOM                                                  0782
  504 FORMAT(10X,'NO FURTHER POINTS WERE FOUND FOR CURVE NO.',I3)       0783
      IF(NPARAM.NE.7) WRITE(6,207) ZSIGN                                0783A
      GO TO 518                                                         0784
  505 WRITE(6,506)                                                      0785
  506 FORMAT(10X,'ONLY ONE POINT WAS FOUND, AND WILL NOT BE PLOTTED',/) 0786
      GO TO 516                                                         0787
  507 J=LOM+50                                                          0788
      IF((ZSIGN.GT.0.0).AND.(JCURVE.GT.0))J=50                          0792
      JCURVE=1                                                          0789
      IF(IGRAPH.LE.-2) GO TO 516                                        0790
      IF((NPARAM.EQ.7).AND.(IPAR7.LT.NPAR7))GO TO 518                   0791
      WRITE(6,511)LOM                                                   0793
  511 FORMAT(/,10X,'THE PREVIOUS POINTS WILL BE PLOTTED AS CURVE ',I2,/) 0794
      IF(NPARAM-7)514,512,514                                           0795
  512 WRITE(6,513)YMAGPK,OMEGPK                                         0796
  513 FORMAT(/,10X,'THE MAXIMUM VALUE OF MAGNITUDE IS',F10.4,' LOCATED A 0797
     &T OMEGA EQUAL TO',F10.4,/)                                        0798
      J=50                                                              0799
  514 CALL DRAW(IP,X,Y,MOD,0,LA(J),ITITLE,XS,YS,IX,IY,2,2,LX,LY,1,LAST) 0800
      WRITE(6,8888) LAST
 8888 FORMAT (//,7X,'LAST =',I3)
      MOD=2                                                             0801
```

```
  516 IP=0
  518 CONTINUE
      IF(NPARAM.EQ.7) GO TO 520
  519 CONTINUE
  520 CONTINUE
      IF((IPAR7.GE.NPAR7).OR.(NPARAM.NE.7)) GO TO 529
C*******GAMMA & PARAM-7 SUPERPOSITION ; SEE INPUT DATA SYMBOL NGAMMA****
  517 IF((NGAMMA.LE.0).OR.(IGAMMA.GT.NGAMMA)) GAMMA=0.0
      IF(IPAR7.GT.1)GO TO 523
      DO 521 I=1,LOE
      OLDHA(I)=HA(I)
      OLDHAT(I)=HAT(I)
      OLDHBT(I)=HBT(I)
  521 OLDHCT(I)=HCT(I)
      DO 522 I=1,MOE
      OLDHD (I)=HD (I)
      OLDHDT(I)=HDT(I)
      OLDHET(I)=HET(I)
  522 OLDHFT(I)=HFT(I)
  523 IPAR7=IPAR7+1
      IF(NGAMMA.GT.0) GO TO 526
      READ(5,5) (HA(I),I=1,LOE)
      READ(5,5) (HAT(I), I=1,LOE)
      READ(5,5) (HBT(I), I=1,LOE)
      READ(5,5) (HCT(I), I=1,LOE)
      READ(5,5) (HD (I), I=1,MOE)
      READ(5,5) (HDT(I), I=1,MOE)
      READ(5,5) (HET(I), I=1,MOE)
      READ(5,5) (HFT(I), I=1,MOE)
      IF(IPAR7.LT.NPAR7) GO TO 200
      DO 525 LOM=1,NLIMIT
      IP=IC(LOM)
      DO 524 I=1,IP
      X(I)=-U(LOM,I)
  524 Y(I)=V(LOM,I)
      J=LOM+1
  525 CALL DRAW(IP,X,Y,MOD,0,LA(J),ITITLE,XS,YS,IX,IY,2,2,LX,LY,1,LAST)
  526 DO 527 I=1,LOE
      HA (I)=OLDHA (I)+GAMMA*GAMHA (I)
      HAT(I)=OLDHAT(I)+GAMMA*GAMHAT(I)
      HBT(I)=OLDHBT(I)+GAMMA*GAMHBT(I)
  527 HCT(I)=OLDHCT(I)+GAMMA*GAMHCT(I)
      DO 528 I=1,MOE
      HD (I)=OLDHD (I)+GAMMA*GAMHD (I)
      HDT(I)=OLDHDT(I)+GAMMA*GAMHDT(I)
      HET(I)=OLDHET(I)+GAMMA*GAMHET(I)
  528 HFT(I)=OLDHFT(I)+GAMMA*GAMHFT(I)
      IF(IGAMMA.LE.NGAMMA) GO TO 26
```

```
  529 IF((NPARAM.NE.7).OR.(IGRAPH.LE.-2)) GO TO 999
C**************************PARAM 7 PHASE CURVES**************************
      IPAR7=2
      IF(IPHASE)537,532,999
  532 DO 536 LOM=1,NLIMIT
      IP=IC(LOM)
      DO 533 I=1,IP
      X(I)=W(LOM,I)
  533 Y(I)=V(LOM,I)
  536 CALL DRAW(IP,X,Y,MOD,0,LA(50),JTITLE,XS,1.,0,IY,2,2,LX,LY,1,LAST)
      WRITE(6,8888) LAST
  537 IPAR7=1
      GO TO 999
C**************************PARAM 7 POLAR PLOT CURVES*******************
  777 MOD=1
      DO 9999 I=1,12
 9999 ITITLE(I)=KTITLE(I)
      DO 542 I=1,8
      RADIUS=I/2.0
      DO 539 J=1,360
      RADIAN=6.28318/360*J
      X(J)=RADIUS*COS(RADIAN)
  539 Y(J)=RADIUS*SIN(RADIAN)
      CALL DRAW(J,X,Y,MOD,0,LA(1),KTITLE,1.,1.,4,4,2,2,8,8,0,LAST)
  542 MOD=2
      DO 543 J=10,180,10
      X(1)=X(J)
      X(2)=-X(1)
      Y(1)=Y(J)
      Y(2)=-Y(1)
  543 CALL DRAW(2,X,Y,2,0,LA(1),KTITLE,1.,1.,4,4,2,2,8,8,0,LAST)
      DO 545 LOM=1,NLIMIT
      IP=IC(LOM)
      DO 544 J=1,IP
      X(J)=S(LOM,J)
  544 Y(J)=T(LOM,J)
      L=LOM+1
  545 CALL DRAW(IP,X,Y,2,0,LA(L),KTITLE,1.,1.,4,4,2,2,8,8,0,LAST)
      WRITE(6,8888) LAST
      IPOLAR=0
  999 IF((MOD).NE.(2)) GO TO 1000
C***********TERMINATING GRAPH LINE FOR EACH PLOT************************
      X(1) = 0.0
      Y(1) = 0.0
      X(2) = (LX-IY)*XS
      Y(2) = 0.0
      CALL DRAW (2,X,Y,3,0,LA(50),ITITLE,1.0,1.0,0,0,2,2,LX,LY,1,LAST)
      WRITE(6,8888) LAST
```

```
      MOD=1                                                                 0893
      IF((IPAR7).EQ.(1)) GO TO 888
  530 DO 531 I=1,12                                                         0894A
  531 ITITLE(I)=JTITLE(I)                                                   0894B
      GO TO 55                                                              0894C
  888 IF((IPOLAR.EQ.1).AND.(NPARAM.EQ.7)) GO TO 777
 1000 CONTINUE                                                              0895
      STOP                                                                  0896
      END                                                                   0897
//LINK.SYSLMOD DD SPACE=(CYL,(5,1,1))
//GO.SYSIN DD *                                                             0899
         1
         1       7
         2       5       1       16       1       1       1       0
         0      50      -1        1       0       0       6      10
   1.       1.      1.      1.
     0.    16.     0.      16.      -10.     50.     .01     10.
  16.      10.     8.       7.       6.      5.     4.5      4.
     3.5    3.     2.5      2.       1.5    1.       .5      0.
    1.
    1.
    1.
 1764.    1260.    225.


 1764.   1260.    1975.    537.5    42.5    1.

         1750.    537.5    42.5     1.
DAVIS,J.P.    CLOSED  LOOP
EXAMPLE #7   MAG  PLCT
DAVIS,J.P.    CLOSED  LOOP
EXAMPLE #7   PHASE  PLOT
```

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13. ABSTRACT

Frequency response techniques are a valuable tool in the analysis and synthesis of linear systems. Extension of these techniques is made to analyze and design systems with a single-valued nonlinear element. The relationship between the characteristics of a nonlinear element and the frequency of the system is developed by simple calculations and a digital computer program.

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